Annales Univ. Sci. Budapest., Sect. Comp. 59 (2026) 95–104

MULTIPLICATIVE FUNCTIONS WHICH ARE ADDITIVE ON SOME AUTOMATIC SEQUENCES

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Communicated by Bui Minh Phong

(Received 20 February 2026; accepted 5 July 2026)

Abstract. Let $k \geq 2$ be an integer. We prove that the 2-automatic sequence $\mathcal{E}$ of numbers with an even number of zeros in their base 2 representation is a k-additive uniqueness set for the set of multiplicative functions. That is, if a multiplicative function f_k satisfies a multivariate Cauchy's functional equation

$$f_k(x_1 + x_2 + \cdots + x_k) = f_k(x_1) + f_k(x_2) + \cdots + f_k(x_k)$$

for arbitrary $x_1, ..., x_k \in \mathcal{E}$, then f_k is the identity function $f_k(n) = n$ for all $n \in \mathbb{N}$. We also show that the Fibonacci-automatic sequence $\mathcal{F}$ of numbers with an odd number of 1's in their Zeckendorf representation is a k-additive uniqueness set for the set of multiplicative functions.

1. Introduction

The functional equation

$$f(x + y) = f(x) + f(y), \quad x, y \in \mathbb{R} \tag{1.1}$$

is called the *Cauchy functional equation*. A function $f : \mathbb{R} \to \mathbb{R}$ satisfying (1.1) is called an *additive function*. In 1821, Cauchy proved that every continuous additive function f is linear, i.e. it is of the form $f(x) = cx$, where c is an arbitrary constant. It is also well known that every locally integrable real additive function is linear. For many years the existence of discontinuous additive functions was an open problem. In 1905, Hamel proved that there exist discontinuous additive functions.

Key words and phrases: Additive uniqueness, multiplicative functions, automatic sequences.
2010 Mathematics Subject Classification: 11A25, 11B85.
This research was supported by ADA University Faculty Research and Development Funds
https://doi.org/10.71352/ac.59.050726

Let $\mathcal{M}$ denote the set of complex valued multiplicative functions, i.e. $f \in \mathcal{M}$ if $f(1) = 1$ and $f(mn) = f(m)f(n)$ whenever $\gcd(m, n) = 1$. In 1992, C. A. Spiro [15] considered the equation (1.1) in the case $f \in \mathcal{M}$ and restricted the domain from $\mathbb{R}$ to $\mathbb{N}$. She showed that if a function $f \in \mathcal{M}$ satisfies (1.1) for all primes x and y, then $f(n) = n$ for all $n \in \mathbb{N}$. More generally, she introduced the following concept.

A set $E \subseteq \mathbb{N}$ is called an *additive uniqueness set* of a set of arithmetic functions F if $f \in F$ is uniquely determined under the condition

$$f(m+n) = f(m) + f(n) \text{ for all } m, n \in E. \tag{1.2}$$

For example, $\mathbb{N}$ and $\{1\} \cup 2\mathbb{N}$ are trivially additive uniqueness sets of $\mathcal{M}$. It is natural to ask about other examples of sparse additive uniqueness sets for $\mathcal{M}$.

Subsequently, several mathematicians showed interest in additive uniqueness sets and Spiro's work has been extended in many directions.

Let $k \geq 2$ be a fixed integer. If there is only one function $f \in F$ satisfying $f(x_1 + x_2 + \cdots + x_k) = f(x_1) + f(x_2) + \cdots + f(x_k)$ for arbitrary $x_i \in E$, $i \in \{1, 2, \ldots, k\}$, then E is called a *k-additive uniqueness set* of F.

In 2010, Fang [5] proved that the set of primes is a 3-additive uniqueness set of $\mathcal{M}$. In 2013, Dubickas and Šarka [4] showed that the set of primes is a k-additive uniqueness set of $\mathcal{M}$ when $k \geq 4$.

In 1999 Chung and Phong [3] showed that the set of positive triangular numbers $T_n = \frac{n(n+1)}{2}$, $n \in \mathbb{N}$ and the set of positive tetrahedral numbers $Te_n = \frac{n(n+1)(n+2)}{6}$, $n \in \mathbb{N}$ are new additive uniqueness sets for $\mathcal{M}$ and Park [11] extended their work to sums of k triangular numbers, $k \geq 3$.

Park [10] proved that the set of nonzero squares is a k-additive uniqueness set of $\mathcal{M}$ for every $k \geq 3$, although it is not a 2-additive uniqueness set (see [2]). In a recent paper [12], Park has showed that the set of positive cubes is a k-additive uniqueness set, like the set of positive squares, for multiplicative functions when $k \geq 3$.

In 2018, B. Kim et al. [8] proved that the set of nonzero generalized pentagonal numbers $\mathcal{P} = \{\frac{n(3n-1)}{2} \mid n \in \mathbb{Z}, n \neq 0\}$, is an additive uniqueness set for $\mathcal{M}$ and the author [6] extended their result to sums of k nonzero pentagonal numbers, $k \geq 3$. Later, B. Kim et al. [9] showed that the set of positive pentagonal numbers and the set of positive hexagonal numbers $\mathcal{H} = \{n(2n-1) \mid n \in \mathbb{N}\}$ are new additive uniqueness sets for the collection of multiplicative functions.

Many examples of k-additive uniqueness sets are asymptotic additive bases of finite order, i.e. sets of integers where every sufficiently large natural number can be written as the sum of at most finitely members from that set. So, it is natural to ask which asymptotic additive bases also have the k-additive uniqueness property.

A set $S \subseteq \mathbb{N}$ is called k-automatic if there exists a deterministic finite automaton M that recognises the language of base k representations of elements of S. Note that a k-automatic set S forms an asymptotic additive basis if and only if S is not sparse and $\gcd(S) = 1$ (see [1]).

The main purpose of this paper is to present examples of two automatic sets that have a nice additive structure and k-additive uniqueness property.

We call a natural number *evening* if the number of zeros in its base 2 representation is even. Let $\mathcal{E}$ be the set of evening numbers, that is,

$$\mathcal{E} = \{1, 3, 4, 7, 9, 10, 12, 15, 16, 19, 21, 22, 25, 26, 28, 31, \ldots\}.$$

It is the sequence A059010 in the On-Line Encyclopedia of Integer Sequences (OEIS).

We can recast the definition in terms of the twisted Thue-Morse sequence

$$\mathbf{t}' = 101001101001\ldots$$

Here t'_n is the sum (mod 2) of zeros in the base-2 representation of n. Thus, n is evening if $t'_n = 0$. From this characterization it easily follows that the evening numbers are 2-automatic.

The Fibonacci numbers are given by the recursive rule $F_i = F_{i-1} + F_{i-2}$ for $i \geq 2$ with the initial values $F_0 = 0$, $F_1 = 1$. The Zeckendorf representation of a number n is a representation

$$n = \sum_{2 \leq i \leq t} e_i F_i,$$

where $e_i \in \{0, 1\}$ and we impose the conditions that $e_t = 1$ and $e_i e_{i+1} = 0$ for $2 \leq i \leq t$, that is, the representation does not contain two consecutive 1's. It is well known that every positive integer has such a representation and this representation is unique.

We can associate a number with the string given by its Zeckendorf representation. The strings accepted by a finite automaton M can be considered as a subset $S = S(M) \subseteq \mathbb{N}$. When an automaton is interpreted in this way, it is called a Fibonacci automaton and the corresponding set is called Fibonacci automatic (see [13]).

We call a natural number *Fibodious* if the number of 1's in its Zeckendorf representation is odd. Let $\mathcal{F}$ be the set of Fibodious numbers, that is,

$$\mathcal{F} = \{1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 25, 27, 28, 30, 31, 32, \ldots\}.$$

It is the sequence A020899 in the On-Line Encyclopedia of Integer Sequences (OEIS).

We can reformulate the definition in terms of the Fibonacci-Thue-Morse sequence

$$\mathbf{ftm} = 01110100100011\ldots$$

Here $\mathbf{ftm}[n]$ is the sum (mod 2) of bits in the Zeckendorf representation of n. Thus, n is Fibodious if $\mathbf{ftm}[n] = 1$. From this characterization we see that the Fibodious numbers are Fibonacci-automatic.

The main results of the paper are as follows.

Theorem 1.1. *Fix $k \geq 2$. If a multiplicative function f_k satisfies*

$$f_k(x_1 + x_2 + \cdots + x_k) = f_k(x_1) + f_k(x_2) + \cdots + f_k(x_k)$$

for arbitrary $x_1, \ldots, x_k \in \mathcal{E}$, then f_k is the identity function.

Theorem 1.2. *Fix $k \geq 2$. If a multiplicative function f_k satisfies*

$$f_k(x_1 + x_2 + \cdots + x_k) = f_k(x_1) + f_k(x_2) + \cdots + f_k(x_k)$$

for arbitrary $x_1, \ldots, x_k \in \mathcal{F}$, then f_k is the identity function.

Note that if we drop multiplicativity condition then $f \equiv 0$ also satisfies (1.1). On the other hand, in [7] the author showed that the 2-automatic set of odious numbers $\mathcal{O}$ with odd number of ones in their base 2 representation is an additive uniqueness set for any set of arithmetic functions with $f(1) = 1$.

2. Preliminaries

In this section, we give some lemmas that will be useful for the proof of the main results. Recall that if $A, B \subseteq \mathbb{N}$, then $A + B$ denotes the sumset $\{a + b : \ a \in A, \ b \in B\}$. The next lemma shows that the set of evening numbers $\mathcal{E}$ and the set of Fibodious numbers $\mathcal{F}$ are asymptotic bases of order 2 for $\mathbb{N}$.

Lemma 2.1.

i) $\mathcal{E} + \mathcal{E} = \mathbb{N} \setminus \{1, 3, 9\}$;

ii) $\mathcal{F} + \mathcal{F} = \mathbb{N} \setminus \{1, 12\}$.

Proof. For the proof we will employ the automatic theorem prover `Walnut` originally designed and written in Java by Hamoon Mousavi [14]. It can verify first-order logic statements on sequences of natural numbers in various numeration systems.

i) We first use the `Walnut` command

```
reg even0 msd_2 "0*1(1*01*0)*1*":
```

which creates an automaton `even0` with a free variable x that evaluates to `TRUE` if and only if x in base-2 has even number of 0's. Here the phrase `msd_2` indicates that we are working in base-2 and $*$ means Kleene closure. Next we use the command

```
def notsum2even "~Ex,y $even0(x) & $even0(y) & x>0
    & y>0 & n=x+y":
```

which computes an automaton for the complement $\mathbb{N} \setminus (\mathcal{E} + \mathcal{E})$. Here `E` is `Walnut`'s abbreviation for the existential quantifier $\exists$, the symbol $\sim$ is logical NOT and & is logical AND. The resulting automaton, depicted in Fig. 1, has 5 states and, by inspection, accepts only 1, 3 and 9.

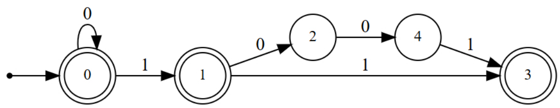

Figure 1. Automaton for $\mathbb{N} \setminus (\mathcal{E} + \mathcal{E})$

ii) We use the `Walnut` command

```
eval sum2f "?msd_fib E i,j (n=i+j) & (FTM[i]=@1) &
    (FTM[j]=@1)":
```

Here ?`msd_fib` evaluates the formula in Zeckendorf representation, `FTM` is a built-in representation of the Fibonacci–Thue–Morse sequence and the expression @`1` represents the constant 1. The resulting automaton, depicted in Fig. 2, has 8 states. A brief inspection shows that this automaton accepts the Zeckendorf representation of all natural numbers, except 1 and 12.

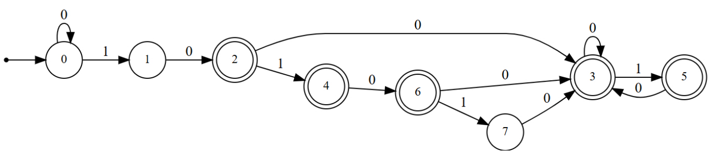

Figure 2. Automaton for sums of two Fibodious numbers

■

Lemma 2.2.

i) *Let $\mathcal{E}_k$ be the set of sums of the k evening numbers. If $k \geq 3$, then $\mathcal{E}_k$ is the set of all positive integers except for $1, 2, \ldots, k-1, k+1$;*

ii) *Let $\mathcal{F}_k$ be the set of sums of the k Fibodious numbers. If $k \geq 3$, then $\mathcal{F}_k$ is the set of all positive integers except for $1, 2, \ldots, k-1$.*

Proof. Lemma 2.1 i) guarantees that every positive integer greater than 9 can be written as the sum of two evening numbers. Thus, every integer greater than 10 can be written as a sum of three evening numbers. It can be easily verified that every positive integer less than or equal to 10 is a sum of three evening numbers except for $1, 2$ and 4. Hence, we can conclude that every positive integer greater than or equal to 5 can be written as a sum of three evening numbers.

It is clear that the sum of k evening numbers can represent k and $k+2$ but cannot represent from 1 to $k-1$. It is also easily checked that the sum cannot represent $k+1$. Since sums of three evening numbers represent all integers greater than or equal to 5, the sum

$$\underbrace{1+\cdots+1}_{k-3 \text{ times}}+x+y+z,$$

where $x, y, z \in \mathcal{E}$, represents all integers greater than or equal to $k+2$.

ii) The proof is similar to i) part. ■

3. Proof of Theorem 1.1

The proof consists of three parts.

Case I: $k=2$. Clearly, $f_2(2)=2$. By Lemma 2.1 i) it is sufficient to determine $f_2(3)$ and $f_2(9)$. Note that $f_2(20)=f_2(4)\cdot f_2(5)=f_2(10+10)= = 4f_2(5)$ or $f_2(5)(4-f_2(4))=0$. Thus, either $f_2(5)=0$ or $f_2(4)=4$. If $f_2(5)=0$, then $f_2(4)=-1$ and from $f_2(8)=f_2(7)+1=2f_2(4)$ we get $f_2(7)=-3$. But this would lead to a contradiction

$$\begin{aligned} f_2(14) = 2f_2(7) = -6 = \\ = f_2(10)+f_2(4) = -1. \end{aligned}$$

Hence, $f_2(4)=4$ and $f_2(3)=f_2(4)-1=3$. Since $f_2(12)=f_2(4)f_2(3)= =f_2(9)+f_2(3)$, we get that $f_2(9)=9$. Thus, f_2 must be the identity function by induction.

Case II: $k=3$. Trivially $f_3(3)=3$. From $f_3(9)=3f_3(3)=2f_3(4)+1$, we get $f_3(4)=4$. Similarly, from $f_3(6)=3f_3(2)=f_3(4)+2$, we obtain $f_3(2)=2$. So, $f_3(n)=n$ for $n\le 4$, and f_3 must be the identity function by Lemma 2.2 i) and induction.

Case III: $k\ge 4$. Let f_k be k-additive on the evening numbers. Trivially $f_k(1)=1$ and $f_k(k)=k$.

Note that

$$\begin{aligned} (k-2)+8 &= (k-2)\cdot 1+4+4 = \\ &= (k-2)\cdot 1+7+1, \\ (k-3)+9 &= (k-3)\cdot 1+3+3+3 = \\ &= (k-3)\cdot 1+7+1+1, \\ (k-3)+26 &= (k-3)\cdot 1+21+4+1 = \\ &= (k-3)\cdot 1+12+7+7. \end{aligned}$$

Let $x=f_k(3)$, $y=f_k(4)$ and $z=f_k(7)$. The above equalities give rise to the system of equations

$$\begin{cases} 2y = z + 1, \\ 3x = z + 2, \\ xz + y + 1 = xy + 2z. \end{cases}$$

The solutions are

$$f_k(3) = f_k(4) = f_k(7) = 1,$$

$$f_k(3) = 3, \quad f_k(4) = 4, \quad f_k(7) = 7.$$

Consider the first solution set $f_k(3) = f_k(4) = f_k(7) = 1$. Arrange the evening numbers into an increasing sequence and let e_n denote the nth term. Then $f_k(e_1) = f_k(e_2) = f_k(e_3) = f_k(e_4) = 1$. By Lemma 2.2 i), every e_n with $n \geq 4$ can be written as a sum of three evening numbers. From the equality

$$\begin{aligned} &(k-4) + 3 + 3 + 1 + e_d = \\ &= (k-4) + 7 + e_a + e_b + e_c \text{ with } a, b, c < d \end{aligned} \tag{3.1}$$

we infer that $f_k(e_n) = 1$ for all $n \geq 5$ inductively.

But for sufficiently large n, e_n can be represented as a sum of k evening numbers. So $f_k(e_n) = k$, which is a contradiction.

Hence, we conclude that $f_k(3) = 3$, $f_k(4) = 4$, and $f_k(7) = 7$. Moreover, (3.1) yields $f_k(e_n) = e_n$ for every $n \geq 1$.

If N is a sum of k evening numbers then $f_k(N) = N$. Otherwise, choose an integer $M \geq k + 2$ such that $\gcd(M, N) = 1$. Then M and MN can be represented as sums of k evening numbers by Lemma 2.2 i). By the multiplicativity of f_k, $M f_k(N) = f_k(M) \cdot f_k(N) = f_k(MN) = MN$. Therefore, $f_k(N) = N$ and this completes the proof. ■

4. Proof of Theorem 1.2

The proof consists of four parts.

Case I: $k = 2$. Clearly $f_2(2) = 2$. Then, $f_2(3) = f_2(2) + 1 = 3$ and $f_2(4) = 2f_2(2) = 4$. So, $f_2(12) = f_2(3)f_2(4) = 12$ and we can conclude that f_2 is the identity function by Lemma 2.1 ii) and induction.

Case II: $k = 3$. Trivially, $f_3(3) = 3$. Observe that $f_3(6) = 3f_3(2) = f_3(3 + 2 + 1) = f_3(2) + 4$ and therefore $f_3(2) = 2$. From Lemma 2.2 ii) and by induction, we conclude that f_3 must be the identity function.

Case III: $k = 4$. Clearly, $f_4(4) = 4$, $f_4(5) = f_4(2 + 1 + 1 + 1) = f_4(2) + 3$ and $f_4(10) = f_4(2)f_4(5) = f_4^2(2) + 3f_4(2)$. On the other hand, $f_4(10) = f_4(5 + 2 + 2 + 1) = 3f_4(2) + 4$. Hence, $f_4^2(2) = 4$ with two solutions

$f_4(2) = -2$ and $f_4(2) = 2$. The first solution yields $f_4(5) = 1$, which leads to the contradiction

$$\begin{aligned} f_4(14) = f_4(5+5+2+2) = -2 = \\ = f_4(2)f_4(7) = (3f_4(2)+1)f_4(2) = 10. \end{aligned}$$

Thus, $f_4(2) = 2$. So, $f_4(n) = n$ for $n \leq 4$, and f_4 is the identity function by Lemma 2.2 ii) and induction.

CASE IV: $k \geq 5$. Let f_k be k-additive on the Fibodious numbers. Trivially $f_k(1) = 1$ and $f_k(k) = k$.

Note that

$$\begin{aligned} (k-2)+6 &= (k-2)\cdot 1 + 3 + 3 = \\ &= (k-2)\cdot 1 + 5 + 1, \\ (k-2)+10 &= (k-2)\cdot 1 + 5 + 5 = \\ &= (k-2)\cdot 1 + 2 + 8, \\ (k-3)+9 &= (k-3)\cdot 1 + 3 + 3 + 3 = \\ &= (k-3)\cdot 1 + 2 + 2 + 5, \\ (k-4)+47 &= (k-4)\cdot 1 + 30 + 8 + 8 + 1 = \\ &= (k-4)\cdot 1 + 40 + 3 + 2 + 2. \end{aligned}$$

Let $a = f_k(2)$, $b = f_k(3)$, $c = f_k(5)$ and $d = f_k(8)$. The above equalities give rise to the system of equations

$$\begin{cases} 2b = c + 1, \\ 2c = a + d, \\ 3b = 2a + c, \\ abc + 2d + 1 = cd + 2a + b. \end{cases}$$

The solutions are

$$f_k(2) = \frac{7}{4}, \quad f_k(3) = \frac{5}{2}, \quad f_k(5) = 4, \quad f_k(8) = \frac{25}{4};$$

$$f_k(2) = f_k(3) = f_k(5) = f_k(8) = 1;$$

$$f_k(2) = 2, \quad f_k(3) = 3, \quad f_k(5) = 5, \quad f_k(8) = 8.$$

Note that $f_k(k+2) = k - 1 + f_k(3)$ and $f_k(k+4) = k - 2 + 2f_k(3)$.

If $3 \nmid (k+2)$, then the equalities

$$\begin{aligned} f_k(3(k+2)) = f_k(\underbrace{3 + \cdots + 3}_{k-3 \text{ times}} + 5 + 5 + 5) = f_k(3)(k-3) + 3f_k(5) = \\ = f_k(3)f_k(k+2) = f_k(3)(k - 1 + f_k(3)) \end{aligned}$$

exclude the first solution set $f_k(2) = \frac{7}{4}$, $f_k(3) = \frac{5}{2}$, $f_k(5) = 4$, $f_k(8) = \frac{25}{4}$.

If $3 \mid (k+2)$, then $\gcd(3, k+4) = 1$ and the equalities

$$f_k(3(k+4)) = f_k(\underbrace{3 + \cdots + 3}_{k-3 \text{ times}} + 8 + 8 + 5) = f_k(3)(k-3) + 2f_k(8) + f_k(5) =$$
$$= f_k(3)f_k(k+4) = f_k(3)(k - 2 + 2f_k(3))$$

exclude the first solution set.

Now, consider the second solution set $f_k(2) = f_k(3) = f_k(5) = f_k(8) = 1$. Arrange the Fibodious numbers into an increasing sequence and let x_n denote the nth term. Then $f_k(x_2) = f_k(x_3) = f_k(x_4) = f_k(x_5) = 1$. By Lemma 2.2 ii), every x_n with $n \geq 4$ can be written as a sum of four Fibodious numbers. From the equality

$$(4.1) \qquad \begin{aligned} &(k-5) + 3 + 2 + 2 + 1 + x_e = \\ &= (k-5) + 8 + x_a + x_b + x_c + x_d \quad \text{with } a, b, c, d < e \end{aligned}$$

we infer that $f_k(x_n) = 1$ for all $n \geq 5$ inductively.

But for sufficiently large n, x_n can be represented as a sum of k Fibodious numbers. So $f_k(x_n) = k$, which is a contradiction.

Hence, we conclude that $f_k(2) = 2$, $f_k(3) = 3$, $f_k(5) = 5$ and $f_k(8) = 8$. Moreover, (4.1) yields $f_k(x_n) = x_n$ for every $n \geq 1$.

If N is a sum of k Fibodious numbers then $f_k(N) = N$. Otherwise, choose an integer $M \geq k$ such that $\gcd(M, N) = 1$. Then M and MN can be represented as sums of k Fibodious numbers by Lemma 2.2 ii). By the multiplicativity of f_k, $Mf_k(N) = f_k(M) \cdot f_k(N) = f_k(MN) = MN$. Therefore, $f_k(N) = N$ and this completes the proof. ■

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