BOOK REVIEW

Ferenc Weisz: *Convergence and Summability of Fourier Transforms and Hardy Spaces*

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The main purpose of this book is to investigate the convergence and summability of one-dimensional and multi-dimensional Fourier transforms. Besides the classical results, recent results of the last 20–30 years are studied. By the well known Carleson's theorem, the Dirichlet integrals $s_T f$ of $f \in L_p(\mathbb{R})$, $1 < p < \infty$, converge to the function a.e. To extend this theorem to $p = 1$, a summability method have to be considered. A well known result in summability theory is Lebesgue's theorem, i.e., the Fejér means of an integrable function converge almost everywhere to the function:

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T s_t f(x)\, dt = f(x) \qquad \text{a.e.}$$

The set of convergence was characterized as the Lebesgue points of f.

The application of the theory of one- and multi-dimensional Hardy spaces in Fourier analysis is an important part of the book. In Chapter 1, one-dimensional Hardy spaces, the Hardy–Littlewood maximal operator and the atomic decomposition are discussed. Using the atomic decomposition, a sufficient condition for an operator to be bounded from the Hardy space to L_p is given. In Chapter 2, some basic facts about one-dimensional Fourier transforms and the Fourier inversion formula are shown. The so-called θ-summation, which contains all well known summability methods, such as the Fejér, Riesz, Weierstrass, Abel, Picard, Bessel, Rogosinski, de La Vallée-Poussin summations is investigated. It is proved that the maximal operators of the summability means

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are bounded from the Hardy space to L_p, whenever $p > p_0$. A weak type inequality and the almost everywhere convergence of the summability means are obtained. The convergence theorem about Lebesgue points mentioned above is proved as well. Using the modern techniques of two- and multi-dimensional summability theorems, simple proofs are given for the strong summability results. After the classical books of Bary and Zygmund, this is the first book which considers strong summability.

In Chapter 3, different types of Hardy–Littlewood maximal operators and multi-dimensional Hardy spaces are introduced. The atomic decomposition of each Hardy space is verified. Sufficient conditions for an operator to be bounded from the Hardy space to L_p are given for each Hardy space.

In Chapters 4 and 5, different summation methods for multi-dimensional trigonometric Fourier transforms are investigated. The integrals in the summability means are taken over the balls of ℓ_q (called ℓ_q-summability) or over rectangles (called rectangular summability). In Chapter 5, it is proved that the maximal operators of the ℓ_q-summability means are bounded from the Hardy space to L_p, whenever $p > p_0$. A weak type inequality is obtained in this case, too, which implies again the almost everywhere convergence. New Lebesgue points are introduced and the convergence at these Lebesgue points are proved for functions from the Wiener amalgam spaces. One of the novelties of this book is that the Lebesgue points are studied also in the theory of multi-dimensional summability. In the last chapter, rectangular θ-summability is investigated and similar results are proved. In this case, two types of convergence and maximal operators are considered, namely, the restricted (convergence over the diagonal or more generally over a cone), and the unrestricted (convergence over $\mathbb{R}^d$).

The book is useful for researchers as well as for graduate or postgraduate students. Especially the first two chapters can be used well by graduate students and the other chapters rather by PhD students and researchers.