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BALL CONVERGENCE OF AN ITERATIVE METHOD FOR NONLINEAR EQUATIONS BASED ON THE DECOMPOSITION TECHNIQUE UNDER WEAK CONDITIONS

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Abstract. In the present paper, we consider convergence analysis of a numerical method considered in Shah and Noor (2015) to solve equations using decomposition technique under weaker assumptions. Using the idea of restricted convergence domains we extend the applicability of this method. Numerical examples where earlier results cannot apply to solve equations but our results can apply are also given in this study.

1. Introduction

Consider the problem of approximating the solution x^* of nonlinear equation

$$F(x) = 0, \tag{1.1}$$

where $F : D \subseteq X \longrightarrow Y$ is a Fréchet-differentiable function and D is a convex set. Due to the wide applications, finding solutions of the equation (1.1) is

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an important problem in mathematics. Many authors considered Newton-like method for obtaining an approximation for the solution x^* of (1.1). Higher order multi-point methods are studied in the literature (see [5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]) for approximating the solution x^* of (1.1).

In the present paper, we consider the following construction considered in [16]

$$\begin{aligned} y_n &= x_n - A_n^{-1}F(x_n), \\ z_n &= y_n - A_n^{-1}F(y_n), \\ x_{n+1} &= z_n - A_n^{-1}F(z_n), \end{aligned} \tag{1.2}$$

where x_0 is an initial point and $A_n = A(x_n) = B(x_n) + F'(x_n)$, $B(x)(.) :$ $: D \longrightarrow L(X,Y)$ is a bounded linear operator for each $x \in D$. If $X =$ $= Y = \mathbb{R}$, $B(x) = \frac{G'(x)}{G(x)}F(x)$, where $G : D \longrightarrow \mathbb{R}$ is a continuous function then, the method reduces to the method considered in [16]. The method in this special case was shown to be efficient for cases when $F'(x^*) \approx 0$.

Our goal is to weaken the assumptions in [16], so that the applicability of the method (1.2) can be extended.

Assumptions of the form

$$\|F'''(x) - F'''(y)\| \leq L\|x-y\|, \;\; x, y \in \Omega, \;\; L \geq 0 \tag{1.3}$$

or

$$\|F'''(x) - F'''(y)\| \leq w(\|x-y\|), \;\; x, y \in \Omega, \tag{1.4}$$

where $w(t)$ is a nondecreasing continuous function for $t > 0$ and $w(0) = 0$ (see [16]) are used in earlier studies such as [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

Example 1.1. A typical function that does not satisfy (1.3) or (1.4) is defined by

$$F(x) = \begin{cases} x^3 \ln x^2 + x^5 - x^4, & x \neq 0 \\ 0, & x = 0, \end{cases} \tag{1.5}$$

where $F : [-\frac{5}{2}, \frac{1}{2}] \longrightarrow \mathbb{R}$. We have that $x^* = 1$,

$$F'(x) = 3x^2 \ln x^2 + 5x^4 - 4x^3 + 2x^2,$$

$$F''(x) = 6x \ln x^2 + 20x^3 - 12x^2 + 10x,$$

and

$$F'''(x) = 6 \ln x^2 + 60x^2 - 24x + 22.$$

Then, obviously, function F''' is unbounded on D. Hence, the results in [16] cannot be used to solve (1.1) using (1.2). We also provide computable error bounds on the distances $\|x_n - x^*\|$, radii of convergence and uniqueness results not given in [16]. In this study, our local convergence is based only on the first Fréchet-derivative. This technique can be used to extend the applicability of other methods [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

The rest of the paper is organized as follows. In Section 2 we present the local convergence analysis. We also provide a radius of convergence, computable error bounds and uniqueness result. Special cases and numerical examples are given in the last section.

2. Ball convergence

The ball convergence of method (1.2) is based on some functions and parameters. Let $w_0, w, v, \beta : [0, +\infty) \longrightarrow (0, +\infty)$ be continuous, non-negative, non-decreasing functions defined on the interval with $w_0(0) = w(0) = \beta(0) = 0$. Define the parameter r_0 by

$$r_0 = \sup\{t \geq 0 : w_0(t) + \beta(t) < 1\}. \tag{2.1}$$

Define functions $g_i, h_i, i = 1, 2, 3$ on the interval $[0, r_0)$ by

$$g_1(t) = \frac{\int_0^1 w((1-\theta)t)d\theta}{1 - w_0(t)} + \frac{\beta(t)\int_0^1 v(\theta t)d\theta}{(1 - w_0(t))(1 - p(t))}.$$

$$g_2(t) = \left(1 + \frac{\int_0^1 v(\theta g_1(t)t)d\theta}{1 - p(t)}\right) g_1(t),$$

$$g_3(t) = \left(1 + \frac{\int_0^1 v(\theta g_2(t)t)d\theta}{1 - p(t)}\right) g_2(t),$$

$$h_i(t) = g_i(t) - 1, \ i = 1, 2, 3,$$

where $p(t) = w_0(t) + \beta(t)$. We have that $h_1(0) = -1 < 0$ and $h_1(t) \to +\infty$ as $t \to r_0^-$. It then follows from the intermediate value theorem that function h_1 has zeros in the interval $(0, r_0)$. Denote by r_1 the smallest such zero. Moreover, we have $h_2(0) = -1 < 0$ and $h_2(r_1) = \frac{\int_0^1 v(\theta r_1)d\theta}{1-p(r_1)} > 0$, since $g_1(r_1) = 1$. Denote by r_2 the smallest zero of function h_2 on the interval $(0, r_1)$. Furthermore, we get that $h_3(0) = -1$ and $h_3(r_2) = \frac{\int_0^1 v(\theta r_2)d\theta}{1-p(r_2)} > 0$, since $g_2(r_2) = 1$. Denote by

r_3 the smallest zero of function h_3 on the interval $(0, r_2)$. Then, we have for each $t \in [0, r_3)$

$$0 \leq g_i(t) < 1, \ \ i = 1, 2, 3. \tag{2.2}$$

Let $U(a, \rho), \bar{U}(a, \rho)$ stand respectively for the open and closed balls in X with center $a \in X$ and of radius $\rho > 0$. Next, we present the local convergence analysis of method (1.2) using the preceding notation.

Theorem 2.1. *Let $F : D \subset X \to Y$ be a continuously Fréchet-differentiable operator. Suppose that there exist $x^* \in D$, non-decreasing continuous functions $w_0, w, v, \beta : [0, +\infty) \longrightarrow [0, +\infty)$ with $w_0(0) = w(0) = \beta(0) = 0$ such that for each $x, y \in D$*

$$F(x^*) = 0, \quad F'(x^*)^{-1} \in L(Y, X), \tag{2.3}$$

$$\|F'(x^*)^{-1}(F'(x) - F'(x^*)\| \leq w_0(\|x - x^*\|), \tag{2.4}$$

$$\|F'(x^*)^{-1}(F'(x) - F'(y)\| \leq w(\|x - y\|), \tag{2.5}$$

$$\|F'(x^*)^{-1}F'(x)\| \leq v(\|x - y\|), \tag{2.6}$$

$$\|F'(x^*)^{-1}B(x)\| \leq \beta(\|x - y\|), \tag{2.7}$$

and

$$\bar{B}(x^*, r_3) \subseteq D, \tag{2.8}$$

where the r_0 is defined by (2.1) and r_3 is the smallest positive zero of function h_3. Then, the sequence $\{x_n\}$ generated for $x_0 \in U(x^*, r_3) - \{x^*\}$ by method (1.2) is well defined in $U(x^*, r_3)$, remains in $U(x^*, r_3)$ for each $n = 0, 1, 2, \ldots$ and converges to x^*. Moreover, the following estimates hold

$$\|y_n - x^*\| \leq g_1(\|x_n - x^*\|)\|x_n - x^*\| \leq \|x_n - x^*\| < r, \tag{2.9}$$

$$\|z_n - x^*\| \leq g_2(\|x_n - x^*\|)\|x_n - x^*\| \leq \|x_n - x^*\|, \tag{2.10}$$

and

$$\|x_{n+1} - x^*\| \leq g_3(\|x_n - x^*\|)\|x_n - x^*\| \leq \|x_n - x^*\|, \tag{2.11}$$

where, the functions $g_i, i = 1, 2, 3$ are defined previously. Furthermore, if there exists $R \geq r_3$ such that

$$\int_0^1 w_0(\theta R) d\theta < 1, \tag{2.12}$$

then the limit point x^* is the only solution of equation $F(x) = 0$ in $D_1 = D \cap \bar{U}(x^*, r_0)$.

Proof. We shall show using mathematical induction that sequence $\{x_n\}$ is well defined, remains in $U(x^*, r_3)$ converges to x^* so that estimates (2.9)–(2.11) are satisfied. By hypothesis $x_0 \in U(x^*, r_3) - \{x^*\}$, (2.1) and (2.4), we get that

$$\|F'(x^*)^{-1}(F'(x_0) - F'(x^*))\| \leq w_0(\|x_0 - x^*\|) \leq w_0(r_0) < 1. \tag{2.13}$$

It follows from (2.13) and the Banach Lemma on invertible operators [1, 15, 17] that $F'(x_0)^{-1} \in L(Y, X)$ and

$$\|F'(x_0)^{-1}F'(x^*)\| \leq \frac{1}{1 - w_0(\|x_0 - x^*\|)}. \tag{2.14}$$

We also have by (2.1), (2.4) and (2.7) that

$$\begin{aligned} \|F'(x^*)^{-1}(A_0 - F'(x^*))\| &\leq \|F'(x^*)^{-1}(F'(x_0) - F'(x^*))\| + \\ &\quad +\|F'(x^*)^{-1}B(x_0)\| \leq \\ &\leq w_0(\|x_0 - x^*\|) + \beta(\|x_0 - x^*\|) = \\ &= p(\|x_0 - x^*\|) \leq p(r_0) < 1, \end{aligned} \tag{2.15}$$

so $A_0^{-1} \in L(Y, X), y_0, z_0, x_1$ are well defined by method (1.2) for $n = 0$ and

$$\|A_0^{-1}F'(x^*)\| \leq \frac{1}{1 - p(\|x_0 - x^*\|)}. \tag{2.16}$$

Notice that from (2.3) and (2.6), we have that

$$\begin{aligned} \left\|F'(x^*)^{-1}F(x_0)\right\| &= \left\|\int_0^1 F'(x^*)^{-1}F'(x^* + \theta(x_0 - x^*))d\theta(x_0 - x^*)\right\| \leq \\ &\leq \int_0^1 v(\theta\|x_0 - x^*\|)d\theta\|x_0 - x^*\|. \end{aligned} \tag{2.17}$$

Using (2.1), (2.2)(for $i = 2$), (2.5)-(2.8), (2.14), (2.16) and (2.17), we get in turn that

$$\begin{aligned} \|y_0 - x^*\| &\leq \|x_0 - x^* - F'(x_0)^{-1}F(x_0) + (F'(x_0)^{-1} - A_0^{-1})F(x_0)\| \leq \\ &\leq \|x_0 - x^* - F'(x_0)^{-1}F(x_))\| + \\ &\quad +\|F'(x_0)^{-1}F'(x^*)\|\|F'(x^*)^{-1}(A_0 - F'(x_0)\| \times \\ &\quad \times\|A_0^{-1}F'(x^*)\|\|F'(x^*)^{-1}F(x_0)\| \leq \end{aligned}$$

$$\leq \frac{\int_0^1 w((1-\theta)\|x_0-x^*\|)d\theta\|x_0-x^*\|}{1-w_0(\|x_0-x^*\|)} +$$

$$+\frac{\beta(\|x_0-x-^*\|)\int_0^1 v(\theta\|x_0-x^*\|)d\theta\|x_0-x^*\|}{(1-w_0(\|x_0-x^*\|))(1-p(\|x_0-x^*\|))} =$$

$$= g_1(\|x_0-x^*\|)\|x_0-x^*\| \leq \|x_0-x^*\| < r_3. \tag{2.18}$$

which shows (2.9) for $n=0$ and $y_0 \in U(x^*, r_3$. Moreover, using (2.2) (for $i=2$), (2.6), (2.17) (for $x_0 = y_0$) and (2.18) we obtain that

$$\|z_0-x^*\| \leq \|y_0-x^*\| + \|A_0^{-1}F'(x^*)\|\|F'(x^*)^{-1}F(y_0)\| \leq \tag{2.19}$$

$$\leq \left(1+\frac{\int_0^1 v(\theta\|y_0-x^*\|)d\theta}{1-p(\|x_0-x^*\|)}\right)\|y_0-x^*\| \leq$$

$$\leq g_2(\|x_0-x^*\|)\|x_0-x^*\| \leq \|x_0-x^*\| < r_3, \tag{2.20}$$

which shows (2.10) for $n=0$ and $z_0 \in \bar{B}(x^*, r_3)$. Furthermore, using (2.2) (for $i=3$), (2.16), (2.17) (for $x_0 = z_0$) and (2.20), we get that

$$\|x_1-x^*\| \leq \|z_0-x^*\| + \|A_0^{-1}F'(x^*)\|\|F'(x^*)^{-1}F(z_0)\| \leq$$

$$\leq \left(1+\frac{\int_0^1 v(\theta\|z_0-x^*\|)d\theta)}{1-p(\|x_0-x^*\|)}\right)\|z_0-x^*\| =$$

$$= g_3(\|x_0-x^*\|)\|x_0-x^*\| \leq \|x_0-x^*\| < r_3, \tag{2.21}$$

so (2.11) holds for $n=0$ and $x_1 \in B(x^*, r_3)$. By simply replacing x_0, y_0, z_0, x_1 by x_k, y_k, z_k, x_{k+1} in the preceding estimates, we arrive at estimates (2.9) – (2.11). Then, from the estimates

$$\|x_{n+1}-x^*\| \leq c\|x_k-x^*\| < r, \tag{2.22}$$

where $c = g_3(\|x_0-x^*\|) \in [0,1)$, we deduce that $\lim_{k\to\infty} x_k = x^*$ and $x_{k+1} \in$ $\in U(x^*, r_3)$. Finally to show the uniqueness part, let

$$T = \int_0^1 F'(x^*+\theta(y^*-x^*))d\theta$$

where $y^* \in D_2$ with $F(y^*)=0$. Using (2.13), we obtain that

$$\begin{aligned} \|F'(x^*)^{-1}(T-F'(x^*))\| &\leq \int_0^1 w_0(\theta\|x^*-y^*\|)d\theta \leq \\ &\leq \int_0^1 w_0(\theta R)d\theta < 1, \end{aligned} \tag{2.23}$$

Hence, we have that $T^{-1} \in L(\mathbb{R}, \mathbb{R})$. Then, from the identity $0 = F(y^*) - -F(x^*) = T(y^* - x^*)$, we conclude that $x^* = y^*$. ■

Remarks 2.1. (a) In the case when $w_0(t) = L_0 t, w(t) = Lt$, the radius $r_A = = \frac{2}{2L_0+L}$ was obtained by Argyros in [1] as the convergence radius for Newton's method under condition (2.9)–(2.11). Notice that the convergence radius for Newton's method given independently by Rheinboldt [14] and Traub [17] is defined by

$$\rho = \frac{2}{3L} < r_A.$$

As an example, let us consider the function $f(x) = e^x - 1$. Then $x^* = 0$. Set $D = U(0, 1)$. Then, we have that $L_0 = e - 1 < L = e$, so $\rho = 0.24252961 < < r_A = 0.324947231$.

Moreover, the new error bounds [1] are:

$$\|x_{n+1} - x^*\| \leq \frac{L}{1 - L_0\|x_n - x^*\|}\|x_n - x^*\|^2,$$

whereas the old ones [4, 7]

$$\|x_{n+1} - x^*\| \leq \frac{L}{1 - L\|x_n - x^*\|}\|x_n - x^*\|^2.$$

Clearly, the new error bounds are more precise, if $L_0 < L$. Clearly, we do not expect the radius of convergence of method (1.2) given by r_3 to be larger than r_A.

(b) The local results can be used for projection methods such as Arnoldi's method, the generalized minimum residual method(GMREM), the generalized conjugate method(GCM) for combined Newton/finite projection methods and in connection to the mesh independence principle in order to develop the cheapest and most efficient mesh refinement strategy [1, 2, 3, 4].

(c) The results can be also be used to solve equations where the operator F' satisfies the autonomous differential equation [1, 2, 3, 4]:

$$F'(x) = P(F(x)),$$

where P is a known continuous operator. Since $F'(x^*) = P(F(x^*)) = P(0)$, we can apply the results without actually knowing the solution x^*. Let as an example $F(x) = e^x - 1$. Then, we can choose $P(x) = x + 1$ and $x^* = 0$.

(d) It is worth noticing that method (1.2) are not changing if we use the new instead of the old conditions [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Moreover, for the error bounds in practice we can use the computational order of convergence (COC)

$$\xi = \frac{\ln \dfrac{\|x_{n+2} - x_{n+1}\|}{\|x_{n+1} - x_n\|}}{\ln \dfrac{\|x_{n+1} - x_n\|}{\|x_n - x_{n-1}\|}}, \quad \text{for each } n = 1, 2, \ldots$$

or the approximate computational order of convergence (ACOC)

$$\xi^* = \frac{\ln \dfrac{\|x_{n+2} - x^*\|}{\|x_{n+1} - x^*\|}}{\ln \dfrac{\|x_{n+1} - x^*\|}{\|x_n - x^*\|}}, \quad \text{for each } \; n = 0, 1, 2, \ldots.$$

(e) In view of (2.4) and the estimate

$$\begin{aligned} \|F'(x^*)^{-1}F'(x)\| &= \|F'(x^*)^{-1}(F'(x) - F'(x^*)) + I\| \leq \\ &\leq 1 + \|F'(x^*)^{-1}(F'(x) - F'(x^*))\| \leq 1 + w_0(\|x - x^*\|) \end{aligned}$$

condition (2.6) can be dropped and can be replaced by

$$v(t) = 1 + w_0(t)$$

or

$$v(t) = 1 + w_0(r_0),$$

since $t \in [0, r_0)$.

(f) Let $X = Y = \mathbb{R}$ and

$$G(x) = e^{-\alpha x}. \tag{2.24}$$

Then, we have

$$\|F'(x^*)^{-1}B(x)\| = |\alpha| \int_0^1 v(\theta\|x - x^*\|)\|x - x^*\|d\theta$$

so

$$\beta(t) = |\alpha| \int_0^1 v(\theta t)td\theta. \tag{2.25}$$

3. Numerical examples

Three numerical examples are presented in this section.

Example 3.1. Let $X = Y = \mathbb{R}^3, D = \bar{U}(0,1), x^* = (0,0,0)^T$. Define function F on D for $w = (x,y,z)^T$ by

$$F(w) = (e^x - 1, \frac{e-1}{2}y^2 + y, z)^T.$$

Then, the Fréchet-derivative is given by

$$F'(v) = \begin{bmatrix} e^x & 0 & 0 \\ 0 & (e-1)y+1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Notice that using the (2.9) conditions, we get $w_0(t) = L_0 t, w(t) = Lt, v(t) = 2$, $B(x) = I, \alpha = 0.25, \beta(t)$ is as in (2.25) $L_0 = e - 1, L = e$. The parameters are

$$r_1 = 0.2039,\ r_2 = 0.0918, r_3 = 0.0654.$$

Example 3.2. Let $X = Y = C[0,1]$, the space of continuous functions defined on $[0,1]$ and be equipped with the max norm. Let $D = \overline{U}(0,1)$. Define function F on D by

$$F(\varphi)(x) = \varphi(x) - 5\int_0^1 x\theta\varphi(\theta)^3 d\theta. \tag{3.1}$$

We have that

$$F'(\varphi(\xi))(x) = \xi(x) - 15\int_0^1 x\theta\varphi(\theta)^2\xi(\theta)d\theta, \text{ for each } \xi \in D.$$

Then, we get that $x^* = 0,\ w_0(t) = L_0 t, w(t) = Lt, v(t) = 2, B(x) = I, \alpha = 0.25, \beta(t)$ is as in (2.25) $L_0 = 7.5,\ L = 15$. The parameters for method are

$$r_1 = 0.0592,\ r_2 = 0.0260, r_3 = 0.0181.$$

Example 3.3. Returning back to the motivational example at the introduction of this study, we have $w_0(t) = w(t) = 96.6629073t, \alpha = 0.25, B(x) = I, \beta(t)$ is as in (2.25) and $v(t) = 2$. Then the parameters are

$$r_1 = 0.0068,\ r_2 = 0.10034, r_3 = 0.0024.$$

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