

## IMAGE RETRIEVAL BASED ON BINARY SIGNATURE AND S-*k*GRAPH

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*Dedicated to András Benczúr on the occasion of his 70th birthday*

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**Abstract.** In this paper, we introduce an optimum approach for querying similar images on large digital-image databases. Our work is based on RBIR (region-based image retrieval) method which uses multiple regions as the key to retrieve images. This method significantly improves the accuracy of queries. However, this also increases the cost of computing. To reduce this expensive computational cost, we implement a binary signature encoder which maps an image to its identification in binary. In order to speed up the lookup, binary signatures of images are classified by the help of S-*k*Graph. Finally, our work is evaluated on COREL's images.

### 1. Introduction

There are three common ways to approach image retrieval [1], including: text-based image retrieval (TBIR), content-based image retrieval (CBIR) and semantic-based image retrieval (SBIR). The text-based image retrieval is difficult and time-consuming to describe image's content. Thus, it is necessary to build a retrieval system by the content of images to find out similar images.

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Furthermore, when querying an image through a key word or an index, the features of images cannot be described visually. So we need to create a method of extracting image's features to find out images with similar content. Extracting visual features of images is an important task of image retrieval process based on content. However, if we retrieve and compare directly the content of the images, then the problem is complicated, time-consuming and costs a lot of storage space. For this reason, when comparing the image's content, we should improve the query speed and save the storage space.

A number of works related to the query image's content have been published recently, such as Extracting image objects based on the change of histogram value [1], Similarity image retrieval based on the comparison of characteristic regions and the similarity relationship of feature regions on images [2], Color image retrieval based on the detection of local feature regions by Harris-Laplace [3], Color image retrieval based on bit plane and  $L^*a^*b^*$  color space [4], Converting color space and building hash table in order to query the content of color images [5], the similarity of the images based on the combination of the image's colors and texture [9], using the EMD distance in image retrieval [10], the image indexing and retrieval technique VBA (Variable-Bin Allocation) based on signature bit strings and S-tree [11], etc.

However, if the method of comparing the similarity of the content is inefficient, the results of querying are the images with content not related to the requested query. The approach of the paper is to create the binary signature of an image. The content of the paper aims to query efficiently "*similarity images*" in a large image database system.

The paper approaches the semantic description of image's content through a binary signature and builds a data structure to store binary signatures. This data structure presents the relationship among the binary signatures as well as image's contents. Based on the description of the semantic relationship of this data structure, the paper finds out the similarity image in content on COREL's image database [6]. The paper devotes the two main sections to reduce the amount of query storage and to speed up image query on the large image database.

**The problem:** *Given an image database  $\mathfrak{S}$ . With each image  $J \in \mathfrak{S}$ , extract the feature region vector  $R^J = (R_1^J, R_2^J, \dots, R_{n_J}^J)$  to describe the visual feature of image. Each feature region  $R_i^J$  is described as a binary signature  $Sig(R_i^J)$ . Each query image  $I$  is an extracted vector of feature region  $R^I = (R_1^I, R_2^I, \dots, R_{n_I}^I)$  which is described as binary signature  $Sig(R^I) = \bigcup Sig(R_i^I)$ . Let  $\phi(I, J) = \phi(R^I, R^J) = g(Sig(R^I), Sig(R^J))$  be a similarity function between the images  $I$  and  $J$ . For this reason, with each query image  $I$  we need to determine a set of image  $Q \subset \mathfrak{S}$  which has the order relation on the basis of similarity measure  $\phi$ .*

To solve the problem, we build a measure which is used to assess the sim-

ilarity between two images and it is called similarity measure. Based on this similarity measure, one order set of similarity images which corresponds to query image is selected. At the same time, based on this order relation, the graph data structure S- $k$ Graph is built to describe the similarity relationship in the contents of images. On the basis of the data structure, the paper proposes an algorithm which creates S- $k$ Graph and a similarity image retrieval algorithm on S- $k$ Graph. In order to illustrate the basic theory, the paper gives examples on a set of COREL images.

The contribution of the paper is an approach to the semantic description of image's content through binary signature as well as building a data structure to store this binary signature. The data structure shows a relationship in the binary signatures which describes the relationship among the contents of images. Based on the description of semantic relationship of this data structure, the paper finds out similarity images which are conformable to content on COREL image database [6].

The paper is organized as follows: **Section 1**, Introduction; **Section 2** presents the construction of theoretical basis of image's binary signature, the similarity measure between images; **Section 3** presents data structure and image retrieval algorithm based on S- $k$ Graph; **Section 4** describes the application and assessing the experimental results of the process of finding similarity images. A conclusion and discussion of future works are given in **Section 5**.

## 2. The similarity measure

According to [7], the binary signature is formed by hashing the data objects, and it has  $k$  1 bits and  $(m - k)$  0 bits in the bit chain  $[1..m]$ , where  $m$  is the length of the binary signature. Data objects and object of the query are encoded with the same algorithm. When the bits in the data object signature are completely covered with the bits in the query signature, then this data object is a candidate of the query. There are three cases: (1) the data object matches the query: each bit in  $s_q$  is covered with the bit in the signature  $s_i$  of the data object (i.e.,  $s_q \wedge s_i = s_q$ ); (2) the object does not match the query (i.e.,  $s_q \wedge s_i \neq s_q$ ); (3) the signatures are compared and then give a *false drop* result.

In order to evaluate the similarity between two images, firstly the paper builds the binary signature to describe the visual features of each image. On the basis of this binary signature, the paper builds similarity measure between two images. The binary signature  $Sig(I)$  of the image  $I$  is defined as follows:

**Definition 2.1.** Let  $F = (F_1, \dots, F_{n_F})$  be a vector to describe the feature values of region  $R_i^I$  of image. Let  $F(R_i^I) = (F_1(R_i^I), \dots, F_{n_F}(R_i^I))$  be a vector value of region feature attribute which is standardized on  $[0, 1]$  (i.e.  $F_j(R_i^I) \in [0, 1]$ ,  $\sum_j F_j(R_i^I) = 1$ ,  $j = 1, \dots, n_F$ ). We set  $B_I^j = b_1^j b_2^j \dots b_m^j$  with  $b_k^j = 1$  if  $k = [F_j(R_i^I) \times m]$ , otherwise  $b_k^j = 0$ ,  $k = 1, \dots, m$ . At that time, the binary signature of feature region  $R_i^I$  is defined as  $Sig(R_i^I) = B_1^j B_2^j \dots B_{n_F}^j$ . The binary signature of image  $I$  is  $Sig(I) = Sig(R^I) = \bigcup_i Sig(R_i^I)$ .

In order to increase the accuracy of image query corresponding to the matching feature regions, we need to match the positions of feature regions between the images. For this reason, we need to determine the center positions of feature regions to match the similarity between the images. The center positions of feature regions is defined as follows:

**Definition 2.2.** Let  $R^I = (R_1^I, R_2^I, \dots, R_{n_I}^I)$  be a vector of feature region of image  $I(x, y)$ . The center of region  $R_i^I \in R^I$  is defined as  $C(R_i^I) = (x_0, y_0) = ((x_s - x_e))$ , so that  $d_E((x_s, y_s), (x_e, y_e)) = \max\{d_E((x_{\alpha i}, y_{\alpha i}), (x_{\alpha j}, y_{\alpha j})) | (x_{\alpha*}, y_{\alpha*}) \in \text{Boundary}(R_i^I)\}$ , where  $d_E$  is an Euclidean distance and  $\text{Boundary}(R_i^I)$  is a boundary of feature region  $R_i^I$ .

On the basis of binary signature and center of feature regions, we set  $R_i^I$  and  $R_j^J$  in turn as the feature regions on the image  $I$  and  $J$ , respectively. At that moment, the distance between two feature regions is defined as follows:

**Definition 2.3.** Let  $R^I = (R_1^I, R_2^I, \dots, R_{n_I}^I)$  and  $R^J = (R_1^J, R_2^J, \dots, R_{n_J}^J)$  be two vectors of feature regions of two images  $I(x, y)$  and  $J(x, y)$ . The distance between feature regions  $R_i^I \in R^I$  and  $R_j^J \in R^J$  is  $\delta(R_i^I, R_j^J) = \|Sig(R_i^I) - Sig(R_j^J)\|_1 + d_E(C(R_i^I), C(R_j^J))$ .

In order to evaluate the correlation between the measures of images, the following theorem shows that the distance  $\delta(R_i^I, R_j^J)$  is a metric.

**Theorem 2.1.** If  $R^I = (R_1^I, R_2^I, \dots, R_{n_I}^I)$  and  $R^J = (R_1^J, R_2^J, \dots, R_{n_J}^J)$  are two vectors of feature regions of two images  $I(x, y)$  and  $J(x, y)$  then the distance  $\delta(R_i^I, R_j^J)$  is a metric.

**Proof.** (1) Suppose that  $R_i^I$  and  $R_j^J$  are two feature regions of  $R^I$  and  $R^J$ . Then,  $\|Sig(R_i^I) - Sig(R_j^J)\|_1 \geq 0$  and  $d_E(C(R_i^I), C(R_j^J)) \geq 0$ . Thus,  $\delta(R_i^I, R_j^J) = \|Sig(R_i^I) - Sig(R_j^J)\|_1 + d_E(C(R_i^I), C(R_j^J)) \geq 0$ . Assume that  $\delta(R_i^I, R_j^J) = \|Sig(R_i^I) - Sig(R_j^J)\|_1 + d_E(C(R_i^I), C(R_j^J)) = 0$ , then  $\|Sig(R_i^I) - Sig(R_j^J)\|_1 = 0$  and  $d_E(C(R_i^I), C(R_j^J)) = 0$ . Furthermore,  $\|\cdot\|_1$  and  $d_E(\cdot, \cdot)$  are the metrics. So,  $Sig(R_i^I) = Sig(R_j^J)$  and  $C(R_i^I) = C(R_j^J)$ . Infer,  $\delta(R_i^I, R_j^J) \geq 0$  and  $\delta(R_i^I, R_j^J) = 0 \Leftrightarrow R_i^I = R_j^J$ .

(2) Let  $k$  be a real number, then:

$$\begin{aligned}
\delta(kR_i^I, kR_j^J) &= \|Sig(kR_i^I) - Sig(kR_j^J)\|_1 + d_E(C(kR_i^I), C(kR_j^J)) \\
&= \sum_{t=1}^{n_F} |kb_t^I - kb_t^J| + \sqrt{(kx_0^I - kx_0^J)^2 + (ky_0^I - ky_0^J)^2} \\
&= |k| \sum_{t=1}^{n_F} |b_t^I - b_t^J| + \sqrt{k^2(x_0^I - x_0^J)^2 + k^2(y_0^I - y_0^J)^2} \\
&= |k| \sum_{t=1}^{n_F} |b_t^I - b_t^J| + |k| \sqrt{(x_0^I - x_0^J)^2 + (y_0^I - y_0^J)^2} \\
&= |k| \left( \sum_{t=1}^{n_F} |b_t^I - b_t^J| + \sqrt{(x_0^I - x_0^J)^2 + (y_0^I - y_0^J)^2} \right) \\
&= |k| \times (\|Sig(R_i^I) - Sig(R_j^J)\|_1 + d_E(C(R_i^I), C(R_j^J))) = |k| \times \delta(R_i^I, R_j^J).
\end{aligned}$$

$$\begin{aligned}
(3) \text{ Let } R^K &= (R_1^K, R_2^K, \dots, R_{n_K}^K) \text{ be a vector of feature regions of image } K, \\
\text{then: } \delta(R_i^I, R_j^J) &+ \delta(R_j^J, R_k^K) = (\|Sig(R_i^I) - Sig(R_j^J)\|_1 + d_E(C(R_i^I), C(R_j^J))) \\
&+ (\|Sig(R_j^J) - Sig(R_k^K)\|_1 + d_E(C(R_j^J), C(R_k^K))) \\
&= (\|Sig(R_i^I) - Sig(R_j^J)\|_1 + \|Sig(R_j^J) - Sig(R_k^K)\|_1) \\
&+ (d_E(C(R_i^I), C(R_j^J)) + d_E(C(R_j^J), C(R_k^K))) \\
&\geq \|Sig(R_i^I) - Sig(R_k^K)\|_1 + d_E(C(R_i^I), C(R_k^K)) = \delta(R_i^I, R_k^K).
\end{aligned}$$

From (1), (2), (3) infer  $\delta(R_i^I, R_j^J)$  is a metric. ■

On the basis of the similarity between the images, the paper builds the similarity measure between two images. On the basis of binary signature and feature regions of image, the similarity measure between two images is defined as follows:

**Definition 2.4.** Let  $R^I = (R_1^I, R_2^I, \dots, R_{n_I}^I)$  and  $R^J = (R_1^J, R_2^J, \dots, R_{n_J}^J)$  be two vectors of feature regions of two images  $I(x, y)$  and  $J(x, y)$ . The similarity function between two images  $I$  and  $J$  is defined as  $\phi(I, J) = \phi(R^I, R^J) = \frac{\|Sig(I) - Sig(J)\| + d_E(C(I), C(J))}{C(I) = (1/n_I) \sum_i C(R_i^I)}$ .

**Lemma 2.1.** The similarity function  $\phi(I, J)$  between two images  $I$  and  $J$  is a metric.

**Proof.** Similar to Theorem 2.1 ■

The process of similarity image retrieval is to find a set of images which have contents similar to the content of the query image. On the basis of the similarity measure at Definition 2.4, with each query image  $I$ , a set of similarity image  $Q$  is defined as follows:

**Definition 2.5** (Similarity Image Retrieval). Let  $\mathfrak{R}_I = \{J_i^I | (J_i^I \in \mathfrak{S}) \wedge (\phi(I, J_i^I) \leq \phi(I, J_j^I) \Leftrightarrow J_i^I \succ J_j^I) \wedge (i \neq j) \wedge (i, j = 1, \dots, n)\}$  be an ordered set based on the measure  $\phi$ . A set of similarity images  $Q \subset \mathfrak{S}$  including  $k$  similarity images is defined as  $Q = \{J_i \in \mathfrak{S} | \phi(I, J_i) = \phi(R^I, R^J) \leq \theta(R^I, R^J), \forall J \in \mathfrak{S}, i = 1, \dots, |Q|\}$ , where  $\theta(R^I, R^J)$  is the threshold of  $\phi(R^I, R^J)$ .

After querying similarity images based on the similarity measure  $\phi$ , we need to rank the query results according to the similarity measure with the query image. Therefore, a set of results including similarity images  $Q$  must be ranked on the similarity measure  $\phi$ . The following theorem shows that a set of result images  $Q$  is an ordered set.

**Theorem 2.2.** *If  $I$  is the query image, then the set of similarity images  $Q \subset \mathfrak{I}$  is an ordered set on the relation  $\succ$ .*

**Proof.** (1) *Symmetry:* If  $I$  is the query image and  $J \in Q$  is an arbitrary image, then  $\phi(I, J) = \phi(I, J)$ , i.e. satisfies the condition  $\phi(I, J) \leq \phi(I, J)$ . Hence,  $J \succ J$ , i.e.  $Q$  has the symmetry on  $\succ$ .

(2) *Antisymmetry:* Given  $J_i, J_j \in Q$  with  $i \neq j$ . Suppose that  $J_i \succ J_j$  (i.e.  $\phi(I, J_i) < \phi(I, J_j)$ ) and  $J_i \neq J_j$  as a result of  $\phi(I, J_i) < \phi(I, J_j)$ . In addition, Lemma 2.1 shows that  $\phi$  is a metric. Therefore, we have  $\phi(I, J_j) \not\leq \phi(I, J_i)$ . So, if  $J_i \succ J_j$ , then  $J_j \not\succ J_i$ . Hence,  $Q$  has an antisymmetry on  $\succ$ .

(3) *Transitivity:* Let  $J_1, J_2, J_3 \in Q$  be three images corresponding to image query  $I$ , suppose that  $J_1 \succ J_2$  and  $J_2 \succ J_3$ , i.e.  $\phi(I, J_1) \leq \phi(I, J_2)$  and  $\phi(I, J_2) \leq \phi(I, J_3)$ . Otherwise, pursuant to Lemma 2.1,  $\phi$  is a metric, so  $\phi(I, J_1) \leq \phi(I, J_3)$ .

Infer: if  $J_1 \succ J_2$  and  $J_2 \succ J_3$  then  $J_1 \succ J_3$ , i.e.  $Q$  has transitivity on  $\succ$ .

From (1), (2), (3) we infer the set of similarity images  $Q \subset \mathfrak{I}$  is an order set on the relation  $\succ$ . ■

### 3. The data structure and image retrieval algorithm

#### 3.1. The S- $k$ Graph

After creating the binary signature and similarity measure between the images, the problem is how to query quickly and reduce the query storage. So, we have to build a data structure to store the binary signatures. We also describe the relationship between the images simultaneously. The paper builds the graph structure to describe the similarity relationship based on the binary signature (Definition 2.1) and the similarity measure (Definition 2.4). This graph structure is called *signature graph* (SG) with each vertex in the graph including the pair of identification  $oid_I$  and signature  $sig_I$  corresponding to the image  $I$ . The weight between two vertices is the similarity measure  $\phi$ . The data structure SG is defined as follows:

**Definition 3.1** (*Signature Graph*). *The signature graph  $SG = (V, E)$  is the graph which describes the relationship between the images, where  $V$  is*

the set of vertices  $V = \{\langle oid_I, Sig(R^I) \rangle | I \in \mathfrak{S}\}$  and  $E$  is the set of edges  $E = \{\langle I, J \rangle | \phi(I, J) = \phi(R^I, R^J) \leq \theta(R^I, R^J), \forall I, J \in \mathfrak{S}\}$ ,  $\theta(R^I, R^J)$  is a threshold value and  $\mathfrak{S}$  is an image database. The weight of each edge  $\langle I, J \rangle$  is a measurement function of the similarity  $\phi(I, J) = \phi(R^I, R^J)$ ,

Each vertex  $v \in V$  in  $SG$  determines  $k$  elements which has the nearest similar measurement. However, with a lot of number of images in a large database, it is difficult to determine the set of similar images corresponding to the query image. Therefore, we build the notion of S-kGraph so that each vertex includes the nearest image and call it  $k$ -neighboring image.

With each  $k$ -neighboring image, the paper builds a cluster including similarity images. This cluster represents an item called center cluster. Then, each cluster includes similarity images defined as follows:

**Definition 3.2.** A cluster  $V_i$  has center  $I_i$ , with  $k_i\theta$  as a radius, is defined as follows:  $V_i = V_i(I_i) = \{J | \phi(I_i, J) \leq k_i\theta, J \in \mathfrak{S}, i = 1, \dots, n\}$ ,  $k_i \in N^*$ .

On the basis of clusters, the paper defines the data structure S-kGraph including vertices as clusters and the weight between two vertices as the similarity measure  $\phi$ . The data structure S-kGraph is defined as follows:

**Definition 3.3** (S-kGraph). Let  $\Omega = \{V_i | i = 1, \dots, n\}$  be a set of clusters so that  $V_i \cap V_j = \emptyset, i \neq j$ . The S-kGraph =  $(V_{SG}, E_{SG})$  is the graph with the weight, including a vertex set  $V_{SG}$  and an edge set  $E_{SG}$  which are defined as follows:  $V_{SG} = \Omega = \{V_i | \exists I_{i_0} \in V_i, \forall I \in V_i, \phi(I_{i_0}, I) \leq k_{i_0}\theta, i = 1, \dots, n\}$ ,  $E_{SG} = \{\langle V_i, V_j \rangle | i \neq j, V_i \in V_{SG}, V_j \in V_{SG}, d(V_i, V_j) = \phi(I_{i_0}, J_{j_0})\}$ , where  $d(V_i, V_j)$  is the weight between two clusters and  $\forall I \in V_i, \phi(I_{i_0}, I) \leq k_{i_0}\theta$ .

With each image we need to classify in clusters through the data structure S-kGraph. So, we need to have the rules of distribution in clusters of the S-kGraph. These rules are defined as follows:

**Definition 3.4** (The Rules of Distribution of Image). Let  $\Omega = \{V_i | i = 1, \dots, n\}$  be a set of clusters so that  $V_i \cap V_j = \emptyset, i \neq j$ ,  $I_0$  be an image which needs to distribute in a set of clusters  $\Omega$ ,  $I_m$  be a center of cluster  $V_m$  so that  $(\phi(I_0, I_m) - k_m\theta) = \min\{(\phi(I_0, I_i) - k_i\theta), i = 1, \dots, n\}$ , where  $I_i$  is a center of cluster  $V_i$ . There are three cases as follows:

- (1) If  $\phi(I_0, I_m) \leq k_m\theta$  then the image  $I_0$  is distributed in cluster  $V_m$ .
- (2) If  $\phi(I_0, I_m) > k_m\theta$  then setting  $k_0 = [(\phi(I_0, I_m) - k_m\theta)/\theta]$ , at that time:
  - (2.1) If  $k_0 > 0$  then creating cluster  $V_0$  with center  $I_0$  and radius  $k_0\theta$ , at that time  $\Omega = \Omega \cup \{V_0\}$ .
  - (2.2) Otherwise (i.e  $k_0 = 0$ ), the image  $I_0$  is distributed in cluster  $V_m$  and  $\phi(I_0, I_m) = k_m\theta$ .

For each image must exist a cluster in the  $S$ - $k$ Graph so that the images are classified. Moreover, to avoid the invalid data in clusters, the images are distributed in unique cluster. *Theorem 3.1* and *Theorem 3.2* show the unique distribution.

**Theorem 3.1.** *Given the  $S$ - $k$ Graph  $= (V_{SG}, E_{SG})$ . Let  $\langle V_i, V_j \rangle \in E_{SG}$  and  $I_{i_0}, J_{j_0}$  in turn be a center of  $V_i, V_j$ . At that time,  $d(V_i, V_j) = \phi(I_{i_0}, J_{j_0}) > (k_{i_0} + k_{j_0})\theta$ , with  $\forall I \in V_i, \phi(I_{i_0}, I) \leq k_{i_0}\theta$  and  $\forall J \in V_j, \phi(J_{j_0}, J) \leq k_{j_0}\theta$ .*

**Proof.** We have  $\forall I \in V_i, \phi(I_{i_0}, I) \leq k_{i_0}\theta$  and  $\forall J \in V_j, \phi(J_{j_0}, J) \leq k_{j_0}\theta$ . Therefore, if  $\forall I' \in \text{Boundary}(V_i), \forall J' \in \text{Boundary}(V_j)$  then  $\phi(I_{i_0}, I') = k_{i_0}\theta$  and  $\phi(J_{j_0}, J') = k_{j_0}\theta$ . Moreover, so  $V_{SG} = \Omega$  is a set of non-overlap clusters (i.e.  $V_i \cap V_j = \emptyset$ ) that is  $\phi(I', J') > 0$ . Infer:  $\forall I' \in \text{Boundary}(V_i), \forall J' \in \text{Boundary}(V_j)$  then  $\phi(I_{i_0}, I') + \phi(I', J') + \phi(J_{j_0}, J') > (k_{i_0} + k_{j_0})\theta$ . Otherwise, because  $\phi$  is a metric, so  $\phi(I_{i_0}, I') + \phi(I', J') + \phi(J_{j_0}, J') \geq \phi(I_{i_0}, J_{j_0})$ . And  $\exists I'_0 \in \text{Boundary}(V_i), \exists J'_0 \in \text{Boundary}(V_j)$  so  $\phi(I_{i_0}, I'_0) + \phi(I'_0, J'_0) + \phi(J_{j_0}, J'_0) = \phi(I_{i_0}, J_{j_0})$ . Therefore,  $\phi(I_{i_0}, J_{j_0}) > (k_{i_0} + k_{j_0})\theta$ . ■

**Theorem 3.2.** *If each image  $I$  is distributed in a set of clusters  $\Omega = \{V_i | i = 1, \dots, n\}$ , then it belongs to a unique cluster.*

**Proof.** Assume that an image  $I$  belongs to two clusters  $V_i, V_j$  with  $V_i \neq V_j$  (i.e.  $(I \in V_i) \wedge (I \in V_j)$ ). Let  $I_i, I_j$  be two centers of clusters  $V_i, V_j$ , we have  $\phi(I_i, I) \leq k_i\theta$  and  $\phi(I_j, I) \leq k_j\theta$ . Therefore,  $\phi(I_i, I) + \phi(I_j, I) \leq (k_i + k_j)\theta$ . On the other hand, since  $\phi$  is a metric, we have  $\phi(I_i, I) + \phi(I_j, I) \geq \phi(I_i, I_j)$ . *Theorem 3.1* shows that  $\phi(I_i, I_j) > (k_i + k_j)\theta$ .

We have the contradictory conditions

$$\begin{aligned} \phi(I_i, I) + \phi(I_j, I) &\geq \phi(I_i, I_j) > (k_i + k_j)\theta, \\ \phi(I_i, I) + \phi(I_j, I) &\leq (k_i + k_j)\theta. \end{aligned}$$

For this reason, each image  $I$  is only distributed in a unique cluster. ■

In order to avoid invalid data, the rules of distribution (*Definition 3.4*) must ensure that the image is classified in a unique cluster. *Theorem 3.3*, *Theorem 3.4* and *Theorem 3.5* show this problem.

**Theorem 3.3.** *If the value  $\phi(I, I_m) - k_m\theta \leq 0$  then it only occurs at one unique  $I_m$ .*

**Proof.** Suppose that  $\exists I_0$  is a center of cluster  $C_0 \in \Omega$  so that  $\phi(I, I_0) - k_0\theta \leq 0 \Leftrightarrow \phi(I, I_0) \leq k_0\theta$ , i.e  $I$  belongs to cluster  $C_0$ . Otherwise, according to the assumption,  $\phi(I, I_m) - k_m\theta \leq 0$ , i.e  $I$  belongs to cluster  $C_m \neq C_0$ . It means that  $I$  belongs to two different clusters which contradicts to *Theorem 3.2*, each image  $I$  only belongs to a unique cluster. Thus, the assumption is false. Inferring, if the value is  $\phi(I, I_m) - k_m\theta \leq 0$ , it only occurs at a unique  $I_m$ . ■



**Theorem 3.4.** *If  $\Omega = \{V_i | i = 1, \dots, n\}$  is a set of clusters and  $I$  is an image then there exists a cluster  $V_{i_0} \in \Omega$  so that  $I \in V_{i_0}$ .*

**Proof.** According to *Definition 3.4*, for any image  $I$  also exists a cluster  $V_{i_0} \in \Omega$  so that  $I \in V_{i_0}$ . ■

**Theorem 3.5.** *Each image  $I$  is distributed in a unique cluster  $C_{i_0} \in \Omega$ .*

**Proof.** According to *Definition 3.4*, any image  $I$  also exists a cluster  $V_{i_0} \in \Omega$  so that  $I \in V_{i_0}$ . According to *Theorem 3.2*, any image  $I$  is only distributed in a unique cluster. Inferring, any image  $I$  is distributed in a unique cluster  $C_{i_0} \in \Omega$ . ■

### 3.2. Extracting the feature regions

In order to execute the similarity image retrieval process according to the proposed theory, first we extract the feature regions of the image. The paper presents the method to extract the feature regions based on the interest points on the image. These interest points are extracted with the intensity and Harris-Laplace detector.

In order to extract the visual features of an image, the first step is to standardize the image size. Let Y, Cb, Cr be Intensity, Blue color, Red color, respectively. According to [3], [4], the Gaussian transformation by human's visual system is fulfilled as follows:

$$L(x, y) = \frac{1}{10} [6.G(x, y, \delta_D) * Y + 2.G(x, y, \delta_D) * Cb + 2.G(x, y, \delta_D) * Cr]$$

with

$$G(x, y, \delta_D) = \frac{1}{\sqrt{2\pi} \cdot \delta_D} \cdot \exp \left( \frac{x^2 + y^2}{2 \cdot \delta_D^2} \right).$$

The intensity  $I_0(x, y)$  for color image is calculated according to equation:  $I_0(x, y) = \text{Det}(M(x, y)) - \alpha \cdot \text{Tr}^2(M(x, y))$ , where  $\text{Det}(\bullet)$ ,  $\text{Tr}(\bullet)$  are Determinant and Trace of matrix, respectively.  $M(x, y)$  is a second moment matrix

$$M(x, y) = \delta_D^2 \cdot G(\delta_I) * \begin{bmatrix} L_x^2 & L_x L_y \\ L_x L_y & L_y^2 \end{bmatrix},$$

where  $\delta_I, \delta_D$  are the integration scale and differentiation scale, and  $L_\alpha$  is the derivative computed the  $\alpha$  direction. The interest points of color image are extracted according to the formula:  $I_0(x, y) > I_0(x', y')$ , with  $x', y' \in A$ ,  $I_0(x, y) \geq \theta$ , where  $A$  is the neighboring of point  $(x, y)$  and  $\theta$  is a threshold value.

Let  $O_I = \{o_I^1, o_I^2, \dots, o_I^n\}$  be a set of feature circles with centers as interest points and a set of feature radiuses  $R_I = \{r_I^1, r_I^2, \dots, r_I^n\}$ . Values of feature radius are extracted with LoG method (Laplace-of-Gaussian) and their value in  $[0, \min(M, N)/2]$ , where  $M, N$  are the height and the width of image. For



Figure 1. A sample result of extracting feature region

each image, the process of extraction interest points is described as follows:

*Step 1.* Convert from RGB color space to YCbCr color space.

*Step 2.* Perform Gaussian transform for the human visual system to calculate the  $L(x, y)$ .

*Step 3.* Calculate the feature intensity  $I_0(x, y)$  for color images. Then, collect the set of interest points.

*Step 4.* Implement the extraction feature regions  $O_I = \{o_I^1, o_I^2, \dots, o_I^n\}$  based on the interest points.

### 3.3. Binary signature of the image

After extracting the feature regions of images, we need to create the binary signatures to describe them. On the basis of the binary signatures, we perform the similarity image retrieval process for the proposed theory.

With each feature region  $o_I^i \in O_I$  of the image  $I$ , the histogram is calculated on the basis of the standard color range  $C$ . Effective clustering method relies on Euclidean measure in RGB color space classifying colors of every pixel on the image. Let  $p$  be a pixel of image  $I$  which has a color vector in RGB as  $V_p = (R_p, G_p, B_p)$ ,  $V_m = (R_m, G_m, B_m)$  be a color vector of a set of standard color range  $C$ , so as  $V_m = \min\{\|V_p - V_i\|, V_i \in C\}$ . At that time, the pixel  $p$  is standardized in accordance with color vector  $V_m$ . According to experiment, the paper uses the standard color range on MPEG7 to calculate histogram for color images on COREL database.

Setting  $o_I^i \in O_I$  ( $i = 1, \dots, N$ ) a feature circle of the image  $I$ , the histogram vector of the circle  $o_I^i$  is  $H(o_I^i) = \{H_1(o_I^i), \dots, H_n(o_I^i)\}$ . Setting  $h_k(o_I^i) = \frac{H_k(o_I^i)}{\sum_j H_j(o_I^i)}$ , a standard histogram vector is  $h(o_I^i) = \{h_1(o_I^i), \dots, h_n(o_I^i)\}$ . Then, the binary signature describes  $h_k(o_I^i)$  as  $B_I^k = b_I^1 b_I^2 \dots b_I^m$ , with  $b_I^j = 1$  if  $j =$

$\lceil (h_j(o_I^i) + 0.05) \times m \rceil$ , otherwise  $b_I^j = 0$ . So, the signature describes the feature region  $o_I^i \in O_I$  as  $Sig(o_I^i) = B_I^1 B_I^2 \dots B_I^n$ . For this reason, the binary signature of the image  $I$  is  $S_I = \bigcup_{i=1}^N Sig(o_I^i)$ . The process of creating of binary signatures for color images is described as follows:

*Step 1.* Calculate the histogram vector  $H(o_I^i) = \{H_1(o_I^i), H_2(o_I^i), \dots, H_n(o_I^i)\}$  on the base of feature region  $o_I^i \in O_I$  with the set of standard color  $C$ .

*Step 2.* For each the feature region  $o_I^i \in O_I$ , standardize histogram vector as  $h(o_I^i) = \{h_1(o_I^i), h_2(o_I^i), \dots, h_n(o_I^i)\}$ .

*Step 3.* Create the binary signature for  $h_k(o_I^i)$  as  $B_I^k = b_I^1 b_I^2 \dots b_I^m$ , with  $b_I^j = 1$  if  $j = \lceil (h_j(o_I^i) + 0.05) \times m \rceil$ , otherwise  $b_I^j = 0$ . The signature describes the feature region  $o_I^i \in O_I$  as  $Sig(o_I^i) = B_I^1 B_I^2 \dots B_I^n$ .

*Step 4.* Create the binary signature of image  $I$  as  $S_I = \bigcup_{i=1}^N Sig(o_I^i)$ .

### 3.4. Creating S-*k*Graph

On the basis of the similarity measure  $\phi$ , the S-*k*Graph is shown in *Definition 3.3* and the rules of distribution of image are shown in *Definition 3.4*, the paper proposes the algorithm to create the data structure S-*k*Graph. With the input image database  $\mathfrak{I}$  and the threshold  $k\theta$ , we need to return the S-*k*Graph. Firstly, we initialize the set of vertices  $V_{SG} = \emptyset$  and initialize the set of edges  $E_{SG} = \emptyset$ , after that create the first cluster. With each image  $I$  we evaluate the distance  $\phi$  with the center of the cluster and to find out the nearest cluster according to  $(\phi(I, I_0^m) - k_m\theta) = \min\{(\phi(I, I_0^i) - k_i\theta), i = 1, \dots, n\}$ . If the condition  $\phi(I, I_0^m) \leq k_m\theta$  is satisfied, the image  $I$  is distributed in cluster  $V_m$ . Otherwise, we consider the rules of distribution as shown in *Definition 3.4* to classify the image  $I$  into appropriate cluster. This algorithm is as follows:

**Algorithm 1.** Create the S-*k*Graph

**Input:** Image database  $\mathfrak{I}$  and threshold  $k\theta$

**Output:** S-*k*Graph =  $(V_{SG}, E_{SG})$

- 1:  $V_{SG} = \emptyset$ ;  $E_{SG} = \emptyset$ ;  $k_I = 1$ ;  $n = 1$ ;
- 2: **for**  $\forall I \in \mathfrak{I}$  **do**
- 3:   **if**  $V_{SG} = \emptyset$  **then**
- 4:      $I_0^n = I$ ;  $r = k_I\theta$ ;
- 5:     Initialize cluster  $V_n = \langle I_0^n, r, \phi = 0 \rangle$ ;
- 6:      $V_{SG} = V_{SG} \cup V_n$ ;
- 7:   **else**
- 8:      $(\phi(I, I_0^m) - k_m\theta) = \min\{(\phi(I, I_0^i) - k_i\theta), i = 1, \dots, n\}$
- 9:     **if**  $\phi(I, I_0^m) \leq k_m\theta$  **then**

```

10:          $V_m = V_m \cup \langle I, k_m \theta, \phi(I, I_0^m) \rangle;$ 
11:     else
12:          $k_I = \lceil (\phi(I, I_0^m) - k_m \theta) / \theta \rceil;$ 
13:         if  $k_I > 0$  then
14:              $I_0^{n+1} = I; r = k_I \theta;$ 
15:             Initialize cluster  $V_{n+1} = \langle I_0^{n+1}, r, \phi = 0 \rangle;$ 
16:              $V_{SG} = V_{SG} \cup V_{n+1};$ 
17:              $E_{SG} = E_{SG} \cup \{ \langle V_{n+1}, V_i \rangle | \phi(I_0^{n+1}, I_0^i) \leq k \theta, i = 1, \dots, n \};$ 
18:              $n = n + 1;$ 
19:         else
20:              $\phi(I, I_0^m) = k_m \theta;$ 
21:              $V_m = V_m \cup \langle I, k_m \theta, \phi(I, I_0^m) \rangle;$ 
22:         end if
23:     end if
24: end if
25: end for
26: Return S- $k$ Graph =  $(V_{SG}, E_{SG});$ 

```

### 3.5. Image retrieval algorithm

After creating the S- $k$ Graph, we need to query the similarity images on it. With each query image  $I_Q$ , we need to query the set of the similarity images  $IMG$ . This query process finds out the nearest cluster in S- $k$ Graph with  $\phi_{\min} = \phi(I_Q, I_0^m) = \min\{\phi(I_Q, I_0^i), i = 1, \dots, n\}$ . On the other hand, we need to query the similarity images at adjacent vertex with the measure less than threshold  $k\theta$ . This algorithm is described as follows:

**Algorithm 2.** Image Retrieval Algorithm based on S- $k$ Graph

**Input:** query image  $I_Q$ , S- $k$ Graph= $(V_{SG}, E_{SG})$ , threshold  $k\theta$

**Output:** set of a similarity image  $IMG$

```

1:  $IMG = \emptyset; V = \emptyset;$ 
2:  $\phi_{\min} = \phi(I_Q, I_0^m) = \min\{\phi(I_Q, I_0^i), i = 1, \dots, n\};$ 
3: for  $V_i \in V_{SG}$  do
4:     if  $\phi(I_0^m, I_0^i) \leq k \theta$  then
5:          $V = V \cup V_i;$ 
6:     end if
7: end for
8: for  $V_j \in V$  do
9:      $IMG = IMG \cup \{I_k^j, I_k^j \in V_j, k = 1, \dots, |V_j|\};$ 
10: end for
11: Return  $IMG;$ 

```

## 4. Experiments

### 4.1. Model of image retrieval system

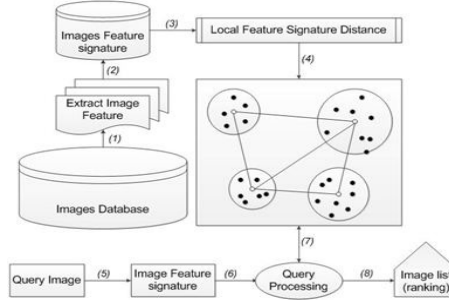


Figure 2. The model of RBIR using S- $k$ Graph

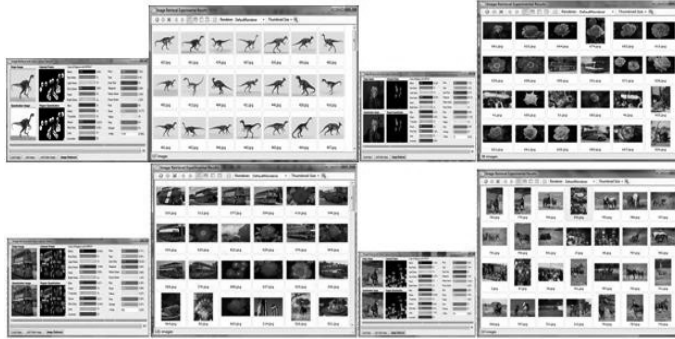


Figure 3. A sample result of image retrieval based on S- $k$ Graph

#### **Phase 1:** Perform pre-processing

*Step 1.* Extract feature regions of the images in database into the form of feature vectors.

*Step 2.* Convert the feature vectors of the image into the form of binary signatures.

*Step 3.* Calculate the similarity measure among the binary signatures of the images and insert into S- $k$ Graph.

#### **Phase 2:** Implement Query

*Step 1.* For each query image, we extract the feature vector and convert into binary signature.

*Step 2.* Perform the process of binary signature retrieval on S- $k$ Graph to find out the similarity images.

*Step 3.* After creating the similarity images, we carry out an arrangement from high to low and give a list of the images on the basis of the similarity binary signatures.

## 4.2. The experimental results

The experimental processing is done on COREL sample data [6] including 10,800 images which are divided into 80 different subjects. With each query image, we retrieve images on COREL data so as to find out the most similar ones to the query image. Then, we compare to the list of subjects of images to evaluate the accurate method.

Binary signatures are introduced into two forms of query structure including SSF (sequential signature file) and S- $k$ Graph. Fig.6 and Fig.7 describe empirical figures about the similarity image retrieval process on COREL images.

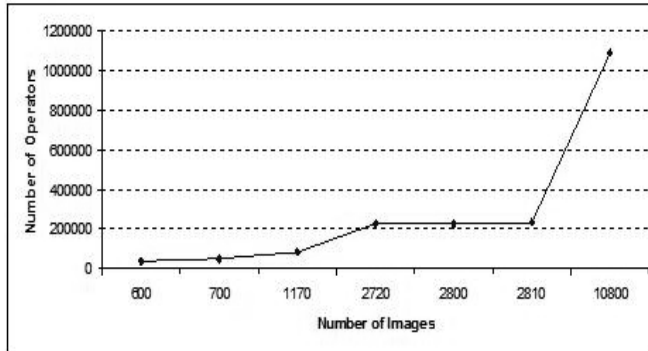


Figure 4. Number of comparisons to create S- $k$ Graph

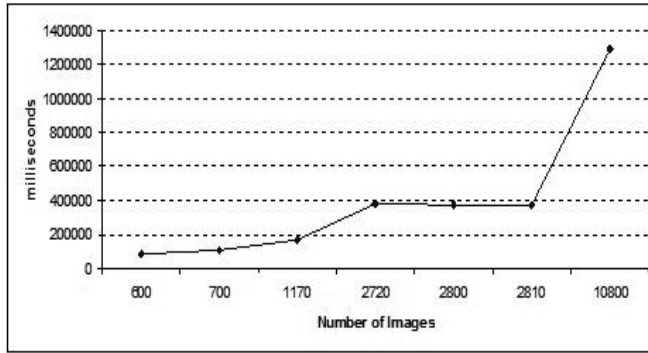


Figure 5. The time to create S-*k*Graph

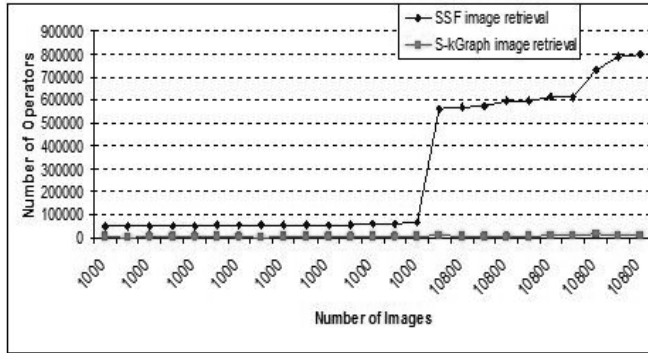


Figure 6. Number of comparisons to query image

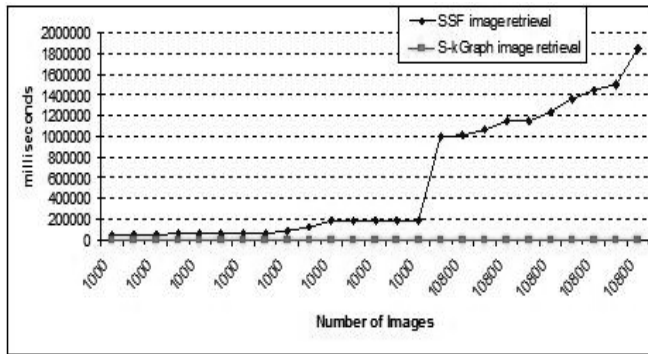


Figure 7. The time to query image

## 5. Conclusion

The paper gives a similar evaluation method between two images on the basis of binary signature and creates S-*k*Graph to describe the relationship

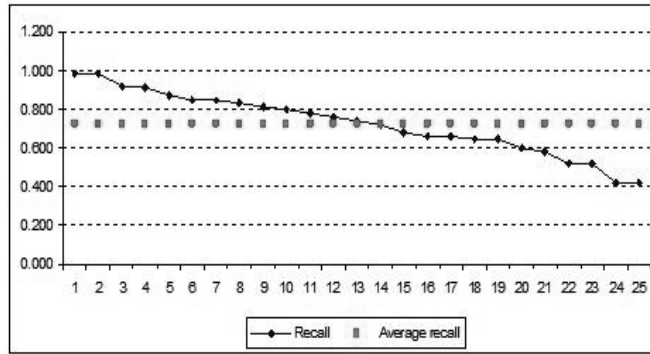


Figure 8. Recall

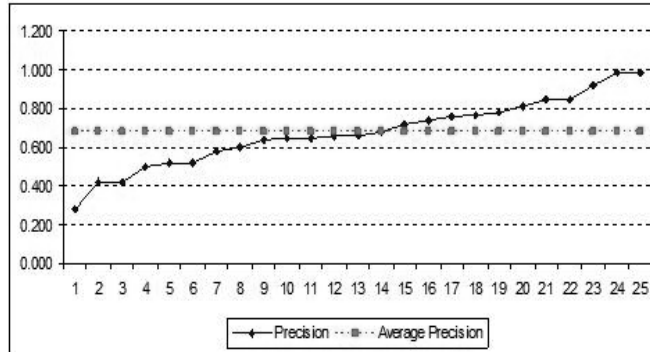


Figure 9. Precision

between images. As a result, the paper creates the image retrieval system model on the basis of feature regions which are to simulate the experiment on COREL's image data classification. According to experimental results, the method of evaluation which is based on S- $k$ Graph speeds up in query similarity images more than query in SSF (sequential signature file). However, the use of the features of color gives an inaccurate result in the sense of image content. Therefore, the future development is to extract objects on the image. Consequently, the paper gives binary signatures to describe objects as well as the contents of images. On the basis of these binary signatures, we assess the similarity measure and return the set of similarity images with query image.



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