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ECG SIGNAL GENERATOR BASED ON GEOMETRICAL FEATURES

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Abstract. Electrocardiograms are widely used in biomedical signal processing to diagnose abnormal heart functioning. Many algorithms have been constructed to analyse, measure and compress these signals. These methods are hard to test because real ECG signals are distorted by several types of noise. In this paper we present an algorithm which generates realistic synthetic ECG signals. This algorithm, among others, can be used for testing new methods in ECG processing.
By using numerical and geometrical parameters, which are of diagnostical importance, the generated signal can be interpreted as a biomedical signal with important diagnostical intervals such as QRS, QT, PR etc. On the other hand this model gives us a strictly mathematical control over the signal. In our interpretation ECG signals are curves with prescribed parameters, including derivatives, curvature etc. We note that Clifford and McSharry and their coauthors [2], [8], [10] carried out a similar program based on a dynamical model. Our approach is essentially different from theirs.

1. Introduction

In medical signal processing the electrocardiogram represents the electrical activity of the heart. In one beat an ECG signal records the potential difference

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between two electrodes that are placed on the surface of the skin. There are several types of recordings called leads which differ in the placements of the electrodes.

The role of digital signal processing in ECG analysis is to help the cardiologists to detect and diagnose diseases. Compressing, transferring and making automated diagnosis of multi-lead recordings by devices with narrow bandwidth is also important especially for those patients who require 24 hours of observation, for instance Holter monitoring.

There are two different ways for testing and comparing these methods. The first one is to use large ECG databases such as Physionet to evaluate the algorithms. Unfortunately the real electrocardiograms are usually affected by noise, so it is difficult to validate the accuracy of the tested procedures. However, using synthetic realistic ECG signals can be advantageous, especially when we want to compare different signal processing techniques. In contrast with the real ECG signals, we can apply and evaluate our methods either on realistic and noise free, or on distorted signals. Furthermore we can avoid any assistance of cardiologists at the early phase of the research.

In the present work we generate signals similar to those corresponding to lead II., which is the voltage between the right arm and the left leg. By taking advantage of separating diagnostical and geometrical properties, we may construct a model that is flexible and well parameterized by each heartbeat.

2. ECG morphology

An ECG signal can be segmented into heartbeats. Each heartbeat consists of five standard waves labelled with the letters P, Q, R, S, T. These waves indicate the depolarisation and the repolarisation phases of heart muscles. Besides, five more interwave timings called PR, PR segment, QRS, QT, ST segment are used. These intervals are indicated on Figure 1. Now we give a brief introduction about the role of these clinical features:

P wave: P waves are usually a low-amplitude feature that represent the depolarization of the atria prior to atrial contraction. They are hard to detect, but important to distinguish various cardiac arrhythmias.

QRS complex: It reflects the depolarization of the ventricles. This is the most significant wave of the ECG due to the large muscle mass of the ventricles. So it can be easily detected and often used to determine the heart rate.

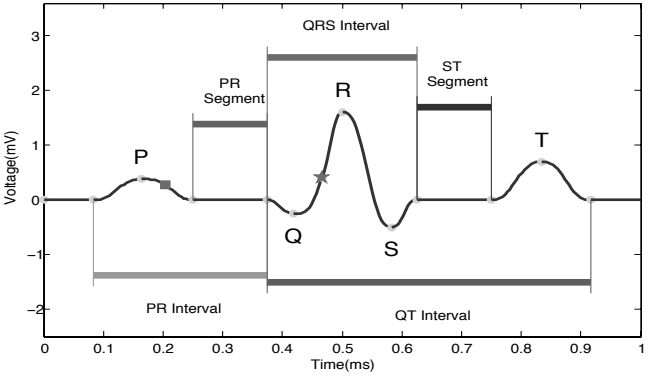

Figure 1. Main characteristics of an ECG curve

T wave: The T wave represents the repolarization of the ventricles. It is a recovery phase of the cardiac muscle. The shape of this wave carries a lot of information about cardiac abnormalities. So it is important to analyse its geometrical properties such as symmetry, asymmetry and slope.

PR interval: It is the time elapsing between the beginning of the P wave and the beginning of the next QRS complex. It reflects conduction through the AV node[1].

PR segment: The PR segment is the flat, usually isoelectric segment between the end of the P wave and the start of the QRS complex. Most of the delay in the PR segment occurs in the AV node.

ST segment: Represents the period from the end of ventricular depolarization to the beginning of ventricular repolarization. ST level shifts are significant markers of cardiac abnormalities.

QT interval: It represents the time between the start of ventricular depolarization and the end of ventricular repolarization. The QT interval is inversely proportional to heart rate: shortens at faster heart rates and lengthens at slower heart rates.

[1]There are two important nodes in the heart, the Sinoatrial and the Atrioventricular node. The former is the physiological pacemaker of the heart that is responsible for triggering the heartbeat. The latter is a part of the electrical control system of the heart which has to delay the cardiac pulses, so that the atria can contract completely before the ventricles begin contracting.

There is one more feature the so-called J point that we will use as a control point in our model. The J point has no standard definition. An informal definition in Chapter 10 of [9] states that "it is the point where the QRS ends and the ST segment begins". A more formal definition in [1], [5] says that "it is the first point of the inflection on the upstroke of the S wave". In our model the J point is represented by control point x_9 which is always an inflection point with low(< 0.1) first derivative. Additionally we calculate the length of the ST segment from this point so both definitions are consistent. For more details on the subject we refer the readers to the monographies [1], [9].

3. Mathematical model

In this section we first give a brief overview of the mentioned ECG generator method by Clifford at al [2], [8]. Then we present our mathematical model of the electrocardiogram and define the diagnostical and geometrical parameters of the curve. Finally we show a set of fixed parameter combination that can be used to generate normal and abnormal ECG curves.

One way to synthesize the electrocardiogram is to use dynamical models [8], [10]. In this case the ECG curve represents a trajectory determined by a differential equation related to the appropriate dynamical system. This model has the advantage of well parameterization, robustness and filtering properties. It is also possible to generate long-term recordings such as 24-hour ECGs by taking into account the heart rate variability. The heart rate may be significantly influenced by sympathetic and parasympathetic[2] activities. For instance the heart rate increases during inhalation and decreases during exhalation. This phenomenon is called respiratory sinus arrhythmia (RSA). In a dynamical model RSA can be treated and parameterized as well, making the synthesized electrocardiogram more realistic. For proper algorithms and further analysis see e.g. [8] [10].

Our approach is an approximation method. We employ different kinds of base functions like polynomials or rational functions to model ECG curves. More precisely we use piecewise polynomial approximation based on 15 control points. The piecewise approximation approach has the following benefits:

- The geometrical parameters of the curve can be adjusted from piece to piece.

[2]Sympathetic and parasympathetic systems are the main parts of the autonomic nervous system. The first one is supposed to prepare the human body for stressful times, sudden loads, to generate energy and to pump more blood to the brain, to the muscles and to the lungs. The second one is responsible for storing energy and enhancing digestion.

- The ability to set diagnostical parameters of the ECG irrespectively of its geometrical parameters.

- The ability to use error measures based on different ECG features like diagnostical intervals, amplitudes etc.

3.1. Polynomial approximation

We control one heartbeat from the electrocardiogram through 15 base points. Denoting these points by $x_1, \ldots, x_{15}$, we have to find the spline S that satisfies the following equation:

$$S^{(i)}(x_k) = f^{(i)}(x_k), \quad (k = 1, \ldots, 15;\ i = 0, 1, 2), \tag{3.1}$$

where $f : \mathbb{R} \to \mathbb{R}$ is a time-varying function that represents the ECG curve. It can be easily solved by the classical Hermite interpolation method. The main question to be answered, is how to choose the base points? Of course they must be related with the diagnostical parameters, because they determine the diagnostical intervals and the wave amplitudes of the ECG. We use the standard points of the electrocardiogram characterised by special geometrical properties. On Figure 1 you can see the end points of one segment of a heartbeat, the extremas and the inflection points.

It follows naturally from our model that the endpoints of the diagnostic subintervals and the points of local extrema must be base points. This way 13 control points are set. It turned out that additional base points should be taken if we want to preserve the geometrical properties, such as monotony and curvature of our signal. Then the problem is twofold. On one hand we need to find sufficient proper control points. On the other hand we need to minimize their number in order to keep the model as simple as possible. Formerly, we used a similar method for compressing and smoothing sound waves. Our experience showed that certain inflection points play a key role in this respect. In case of ECG signals a typical inflection point of this type appears in the QRS complex. That point is indicated by a star on Figure 1. Yet another control point is needed because of the possible shapes of the P wave. All of the possible P wave shapes are described in the paper [12] and some of them are presented on Figure 2. This new control point in the P wave is marked by a square on Figure 1. After having set the control points, we must specify the order of interpolation at these points. Based on geometric considerations given above, the values of the function and the first and the second derivatives will be described at the inner control points. So in this model we have 15 base points and 3 values associated with each of them. All these go up to more than 50 parameters which makes it complicated for the first look. However these

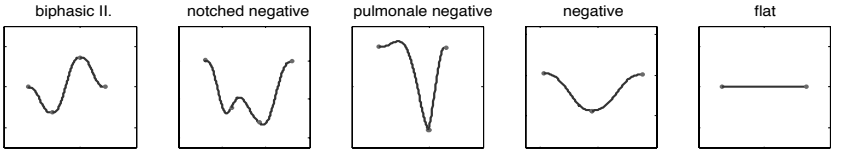

Figure 2. The first five shape models for wave P

parameters can not be chosen arbitrarily if we want to generate a realistic ECG signal. For instance the ECG curves are not affected by noise in our model, so we might as well set the baseline to zero. It means that the values at 8 control points are equal to zero. Similarly, the tangents are zero at all of the 5 extreme points. Moreover, we can set the range of the amplitudes and the interwave timings automatically. We took the data constraints from Section 3.3 in [2]. This way the complexity of the problem can be reduced.

We will distinguish diagnostical and geometric parameters. The 15 diagnostical parameters can be divided into two classes. The first group contains the diagnostical intervals: PR, PR segment, QRS, QT, ST segment. Figure 1 was generated by our program and it shows the onsets and the offsets of each interval related to the base points. The second class includes the positions and the amplitudes of the P, Q, R, S, T waves.

Tangents and curvatures are the geometric parameters. These can be set at some but not all of the base points. For instance the curvature can be set only at the extremas, but the tangents can be varied at all of the non extreme points. At the endpoints, x_1, x_{15}, we demand only interpolation constraints. The variable parameters are summarized in the list below

$$S(x_k) = f(x_k) \qquad (k = 1, 15), \tag{3.2}$$

$$S^{(i)}(x_k) = f^{(i)}(x_k) \qquad (k = 2, 4, 5, 6, 8, 11, 12, 14;\ i = 0, 1), \tag{3.3}$$

$$S^{(i)}(x_k) = f^{(i)}(x_k) \qquad (k = 3, 7, 9, 10, 13;\ i = 0, 2). \tag{3.4}$$

We note that the curvature as a geometric parameter is easy to calculate from the derivatives by the well-known formula

$$\kappa(x_k) = \frac{|f''(x_k)|}{(1 + f'(x_k)^2)^{3/2}} \quad (k = 3, 7, 8, 10, 13), \tag{3.5}$$

where $\kappa(x_k)$ is the curvature of the function f at the base point x_k.

During the construction of our model we have performed several tests using real ECG signals from the physionet database [6]. Then we had to find the proper parameters including some derivatives of first and second orders. The most significant waves such as QRS, T can be easily detected. On the other

hand the derivatives are sensitive to noise for discrete data sequence. Therefore we suggest to use piecewise Hermite interpolation again to approximate the first and the second derivatives. We fit a second order polynomial over 3 adjacent point, $s[n-1]$, $s[n]$ and $s[n+1]$ to compute the derivatives analytically at the middle point. We used the following finite difference formulas:

$$s'[n] = \frac{s[n+1] - s[n-1]}{2} \tag{3.6}$$

$$s''[n] = s[n+1] - 2s[n] + s[n-1]. \tag{3.7}$$

3.2. ECG profiles

Qualifying ECG signal processing methods requires databases with a high amount of test signals. In order to generate such databases we need to vary our parameters. In addition we have to keep the typical geometrical and diagnostical properties related to the actual ECG type. Therefore we define the ECG profile by means of the upper and the lower bounds of the parameters. These bounds are typical to certain kinds of cardiac abnormalities. This way we can define profiles for several types of diseases using their medical statistics. Then we can generate different realistic ECG signals of a certain type by randomizing the parameters between the bounds. All of them carry the same diagnostical information.

As we mentioned above we use 15 diagnostical and 2 geometrical parameters at each control point. So if we want to declare a new profile we should give $15+$ $+15 \times 2$ lower and upper bound pairs. The length of the diagnostical intervals and the amplitudes are given in milliseconds and in millivolts respectively except the amplitudes of the Q and S waves. Namely, these negative deflections are given in the percentage of the amplitude of wave R. The following table shows the parameters of a normal ECG profile.

	P	Q	R	S	T
Amplitudes (mV)	(0.1,0.2)	0.20	(1,2)	0.25	(0.1,0.5)
Positions	(1/2,20)	(1/4,10)	(2/4,10)	(3/4,10)	(1/2,20)
Tangents	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
Curvatures	(-30,0)	(90,100)	(-100,0)	(-10,100)	(-70,0)
	PR	PR seg.	QRS	ST seg.	QT
Intervals (secs)	(120,200)	(50,120)	(80,120)	(80,120)	(300,430)

Table 1. Parameters of the ECG model

In Table 1 the positions in the second row refer to the horizontal alignment of the wave peaks. These values are randomly generated from an I_k $(k = 1, \ldots, 5)$

interval for each wave. The first element of the pair sets the center of each I_k which is related to the right diagnostical interval that contains the wave. For instance, the first number of $(1/2, 20)$ at P means that the center of the I_1 intervals has been set to the middle of the P wave. The second parameter means that the half length of I_1 will be 20 milliseconds. The parameters of the Q, R, S, T peaks are understood in a similar way. We note that at the other control points the tangent and curvature are fixed to be zero in this profile. The correct amplitudes and interwave timings were assessed from Section 3.3 in [2].

4. Results

The proposed algorithm was implemented in MATLAB. We generated two ECG databases for normal and asymmetry tests each with 100 synthesized signals. We give a detailed description about these tests in Section 5.

In order to control the generation process a MATLAB Toolbox was created with an interactive GUI. It consists of 3 view mode each with different functionality:

Geometrical view: It was designed to show the geometrical parameters of the ECG curves. So only tangents and curvatures can be seen at each base point. The positions of these points are expressed with their coordinates.

Diagnostical view: It is a conversation between geometrical and diagnostical properties. User can change the duration of five diagnostical intervals and the amplitudes of the waves. All parameters are given in milliseconds and millivolts. This view mode can be seen on Figure 1.

Graph paper view: It is a standard medical viewpoint. Each ECG recorder prints the output signals to a graph paper which has a regular grid. As usual the time is represented on the x-axis and voltage is represented on the y-axis. In addition, each small square of the grid is 1mm in length and represents 40 ms time and 0.1 mV amplitude.

The parameters are converted proportionally between view modes, so we can examine the same ECG signal from different viewpoints.

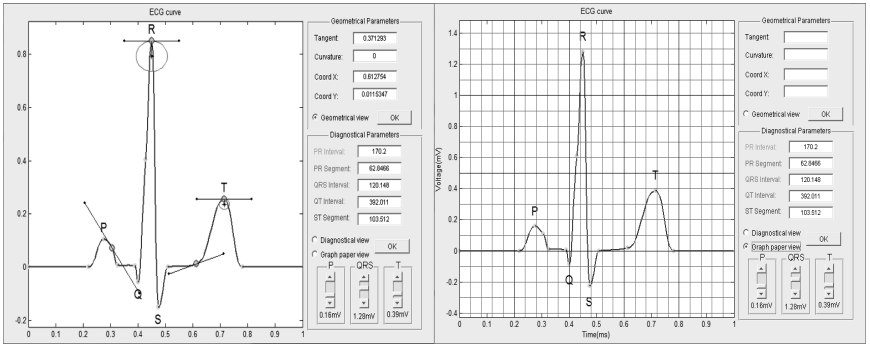

Figure 3. Geometrical view (left) and graph paper view (right) of an ECG curve

5. Experiments

In this section we propose two practical application of our model. First, we share our experience of working with rational functions to compress ECG curves. Second, we introduce how we can apply these results to measure the reconstruction error efficiently.

5.1. Rational function approximation

There is a wide range of applications of rational function systems. For instance we can use them for compressing ECG signals [3]. The process consists of two phases. In phase one we have to find a proper system of rational functions. Phase two is the approximation with respect to the corresponding linear subspace. An effective method in phase one is the Nelder–Mead algorithm [7] for finding the poles of the rational functions in question. However the Nelder–Mead algorithm is not stable in the sense that it may result in different poles for the same problem depending on the initial values. Therefore the Nelder–Mead algorithm for rational approximation of ECG signals needs to be calibrated. Our ECG signal generator can be very useful for this purpose for it produces realistic ECG signals not affected by noise.

The construction of orthogonal rational function systems with given poles goes as follows. The Blaschke functions

$$B_a(z) = \frac{z-a}{1-\overline{a}z} \quad (z \in \mathbb{C}\setminus\{1/\overline{a}\})\,. \tag{5.1}$$

can be used as building blocks. Namely, if $\mathbf{a} = \{a_1, a_2, \ldots, a_n\}$ is a sequence of poles within the unit disc $\mathbb{D}$ then the corresponding orthogonal system is the so-called Malmquist–Takenaka system:

$$\Phi_k^{\mathbf{a}}(z) = \frac{\sqrt{1-|a_k|^2}}{1-\overline{a}_k z} \prod_{j=1}^{k-1} B_{a_j}(z) \quad (z \in \overline{\mathbb{D}}, k = 1, \ldots, n). \tag{5.2}$$

The Malmquist–Takenaka (MT) system turned on to be very efficient for representing and decomposing ECG curves by considering the signal as a time varying periodic function on the unit circle. We refer to [11] for details.

The compression rate depends on the choice of the poles. There are several ways to find the best poles that minimize the global error of the approximation. One of them is the Nelder–Mead algorithm [7]. We tested the stability of the Nelder–Mead algorithm for ECG signals by using the positions and the amplitudes of the P, R, T wave normed peaks as initial values. The testing process consists of four steps:

1. Generate normal ECG curves as rational functions.
2. Apply the Nelder–Mead algorithm to find the poles for the signal generated in step 1.
3. Approximate the original curves with the new poles and coefficients.
4. Compare the original and the approximated curves, poles, and coefficients.

The problem with step one is how to create a realistic ECG curve by pole vectors and coefficients directly. This is when our method comes into picture. Namely, we use our method for generating a realistic ECG curve. Then we use rational approximation to represent it as a rational function. It is clear that the way to control the ECG curve through poles and coefficients would be much harder than to set these parameters explicitly by our model. Accordingly, a normal electrocardiogram was created by our polynomial model and then we applied steps 2-3 to determine an appropriate pole-vector and coefficients for step 1. Now, we have an ECG curve that was generated by rational functions, so we can continue the test with step 2-4.

The results of our tests can be summarized as follows. Figure 4 shows the percentage root mean square difference (PRD) between the original and the approximated poles. The PRD of these poles has been computed by duplicating the middle pole in equation (5.3). As we can see most of the poles has less error than 20%, especially at the second pole. But this small distortion in the positions of the poles radically changes the coefficients which is shown by Figure 5. Our conclusion is that the problem of stability can only be ensured if the initial values are close to the actual poles.

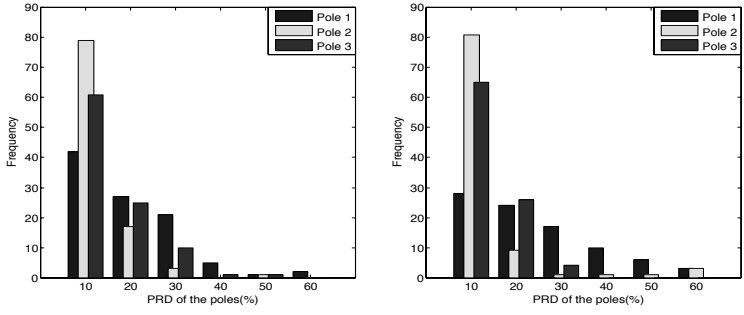

Figure 4. PRD of each pole according to normal (left) and asymmetrical (right) tests

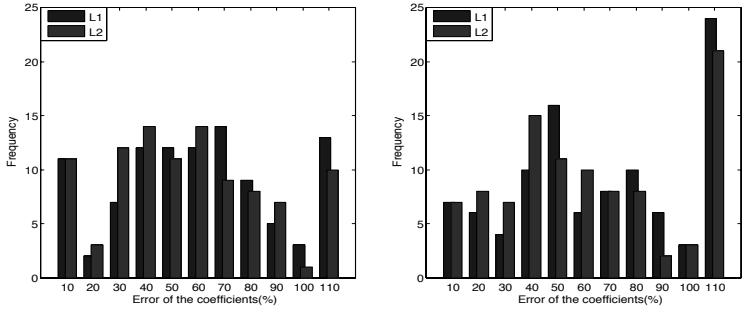

Figure 5. PRD of the coefficient vectors in L_1 and L_2

5.2. Error measures

Comparing ECG signal processing algorithms depends on the measure of the reconstruction error. The PRD is one of the most general error measurement:

$$\text{PRD} = \sqrt{\frac{\sum_{n=1}^{N}(x(n)-\widetilde{x}(n))^2}{\sum_{n=1}^{N}(x(n)-\overline{x})^2}} \times 100 \tag{5.3}$$

where $\overline{x}$ is the signal mean, $x(n)$ is the original signal, $\widetilde{x}(n)$ is the reconstructed signal and N is the length of the window over which the PRD is calculated.

It is easy to calculate this type of error due to its simplicity. But this formula neglects the diagnostical features of the electrocardiogram. On the

other hand, sometimes we need to compare special algorithms such as QRS detector, T wave analyser etc. In this case, the PRD is irrelevant, we are only interested in the error of the given features. Consequently, we have to measure the degree of distortion of the reconstructed signal in terms of the relevant diagnostic information. This is why the so-called Weighted Diagnostic Distortion was introduced:

$$\mathrm{WDD}(\beta, \widehat{\beta}) = \Delta\beta^T \cdot \frac{\Lambda}{\mathrm{tr}[\Lambda]} \cdot \Delta\beta \times 100 \tag{5.4}$$

where $\Delta\beta$ is the normalized difference between the original and the reconstructed feature vectors β,$\widehat{\beta}$ and Λ is a diagonal matrix of weights. For further details see e.g. [12]. Each feature vector consist of 18 elements which are composed of diagnostical intervals, wave amplitudes and wave shapes. Compare to the dynamical model [8] here we can control all the diagnostical features that are needed to compute the WDD. So we can test and compare the performance of the ECG processing methods on one heartbeat by randomizing the relevant parameters. We performed two tests with different kinds of ECG parameters. First we used our method to generate normal electrocardiograms and test the accuracy of the MT reconstruction. In the second case, the sensitivity of the MT system was tested by modifying the following parameters from Table 1.

	P	Q	R	S	T
Positions	(3/4,10)	(2/8,10)	(6/8,10)	(7/8,10)	(1/4,10)
	PR	PR seg.	QRS	ST seg.	QT
Intervals (secs)	(130,170)	(40,40)	(80,120)	(80,120)	(300,430)

Table 2. Asymmetric parameters of the ECG model

Only 3 poles were used to generate the MT system in equation (5.2). However poles can be repeated periodically to make the approximation more accurate. Each test consists of 100 synthesized ECG signals sampled at 256 Hz compressed under different periodicity conditions. Comparison of the original and the reconstructed signals in PRD and WDD can be seen at Figure 6. As we can see the MT reconstruction is more sensitive to the asymmetries of the original signal. But the error can be reduced to an acceptable level ($< 5\%$) by repeating the poles periodically at least 5 times. In this case the compression ratio related to one beat is 36/256 because 15 complex coefficients and 3 complex poles are required for the reconstruction process.

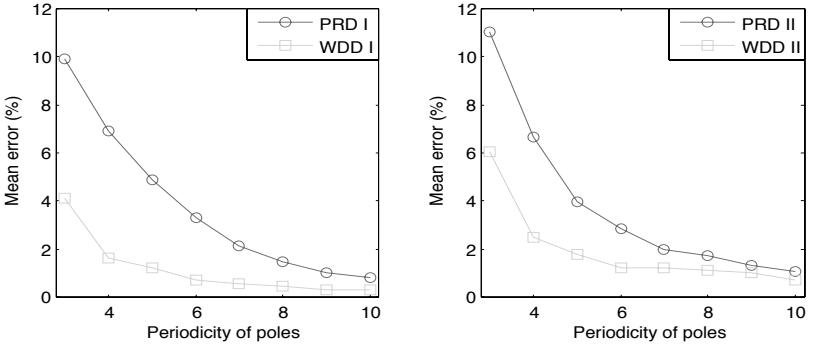

Figure 6. The average PRD and WDD error over 100-100 synthesized signals including normal (left) and asymmetric (right) ECGs

6. Conclusion

We have presented another method for generating electrocardiograms that can be used to replicate many of the important features of the human ECG.

We have shown how to synthesize ECG signals with the same diagnostical but different geometrical parameters and vice versa. In particular, we have demonstrated that the polynomial approximation can serve as an efficient method for ECG modelling. Depending on the positions of the base points, the procedure can be used to compare the accuracy of different signal processing methods by using feature based measures like WDD.

Further research is required on simulating different types of cardiac abnormalities. In this case, it is worth to extend the model by additional base points and new ECG profiles to describe heart diseases. Synthesizing multilead ECG signals is also a very obvious intention since it would make our model more general. A related open question is that of the filtering property, whether or not we can use this model for denoising ECG signals.

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