Annales Univ. Sci. Budapest., Sect. Comp. 27 (2007) 217-228

WAITING TIME IN CYCLIC–WAITING SYSTEMS

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To the memory of M.V. Subbarao

1. Introduction

In [5] we have introduced the so-called cyclic-waiting system functioning in the following way: if the entering entity cannot be serviced upon arrival it joins the queue in which it is cycling with a fixed cycle time, its further requests for service may be put only at the moments differing from the moment of arrival by the multiples of this cycle time T. The service is realized by the FCFS service discipline. There we determined the ergodic distribution for such systems in case of Poisson arrivals and exponentially distributed service time. The model described in [5] received further development in several papers. It was investigated for the case of uniform service time distribution [6], in case of time-limited tasks [8]. Koba [1] found sufficient condition for the existence of equilibrium for the $GI/G/1$ system of this type, Koba and Mykhalevich [2] compared the FCFS and classical retrial service discipline for it. Kovalenko [3] generalized some of Koba's results in case of light traffic. In [7] we studied the model for the discrete time case, namely with geometrically distributed interarrival and with geometrically and uniformly distributed service times.

The original model was raised in connection with the landing process of airplanes, in [4, 9] it appeared as exact model for the transmission of optical signals where because of lack of optical RAM the fiber delay lines (FDL) are used.

The research was supported by the Hungarian-Romanian Intergovernmental Cooperation for Science and Technology under grant RO-40/2005. The first author was partly supported by the Hungarian National Foundation for Scientific Research under grant OTKA K60698/2005.

The quality of functioning of queueing systems may be considered from the viewpoint of the owner of the system and from that of customers, the measures of quality may partly overlap. From the viewpoint of owner the queue length gives the primary information, from the viewpoint of a customer the waiting and sojourn times play essential role. In our previous papers we were dealing with the queue length, other authors started with the waiting time. The two approaches have to meet at the stability condition, intuitively it is clear that the existence of ergodic distribution for the queue length has to imply the finiteness of waiting time.

In [1] Koba found sufficient condition for the stability of $GI/G/1$ cyclic-waiting system and gave the system of equations determining the waiting time's ergodic distribution. In general case one cannot solve it, as example she mentions the situation when the service time is constant. In this paper our purpose is to present this method works for exponentially and geometrically distributed service times and to show that starting with the waiting time we come to the same stability conditions as in [5] and [7].

2. Preliminaires

In [5] we considered the aforementioned cyclic-waiting system when the interarrival and service times were exponentially distributed with parameters λ and μ, respectively. The states were defined as the number of customers in the system at moments just before starting the services of customers, these values constituted a Markov chain. Its matrix of transition probabilities had the form

$$(1)\qquad \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \dots \\ a_0 & a_1 & a_2 & a_3 & \dots \\ 0 & b_0 & b_1 & b_2 & \dots \\ 0 & 0 & b_0 & b_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix},$$

its elements were determined by the generating functions

$$A(z) = \sum_{i=0}^{\infty} a_i z^i = \frac{\mu}{\lambda+\mu} + \frac{\lambda z}{\lambda+\mu} \frac{(1-e^{-\mu T})e^{-\lambda(1-z)T}}{1-e^{-[\lambda(1-z)+\mu]T}},$$

$$B(z) = \sum_{i=0}^{\infty} b_i z^i = \frac{1}{(1-e^{-\lambda T})[1-e^{-[\lambda(1-z)+\mu]T}]} \times$$

$$\times\left\{\frac{1}{2-z}\left(1-e^{-\lambda(2-z)T}\right)\left(1-e^{-[\lambda(1-z)+\mu]T}\right)-\right.$$

$$\left.-\frac{\lambda}{\lambda(2-z)+\mu}\left(1-e^{-[\lambda(2-z)+\mu]T}\right)\left(1-e^{-\lambda(1-z)T}\right)\right\}.$$

We found the equilibrium distribution

$$P(z)=\sum_{i=0}^{\infty}p_iz^i=p_0\frac{(\lambda z+\mu)B(z)-(\lambda+\mu)zA(z)}{\mu[B(z)-z]},$$

$$p_0=1-\frac{\lambda}{\lambda+\mu}\frac{1-e^{-(\lambda+\mu)T}}{e^{-\lambda T}(1-e^{-\mu T})},$$

and the ergodicity condition

$$\frac{\lambda}{\mu}<\frac{e^{-\lambda T}(1-e^{-\mu T})}{1-e^{-\lambda T}}. \tag{2}$$

In [7] we investigated the discrete time version of this system. The cycle time T was divided into n equal parts, for a time slice T/n a customer entered with probability r, so interarrival time was equal to i time slices with probability $(1-r)^{i-1}r$. The service time had geometrical distribution, too, it was equal to i time slices with probability $(1-q)^{i-1}q$. The matrix of transition probabilities had the same form as for continuous time (1), the elements of matrix were given by the generating functions

$$A(z)=\sum_{i=0}^{\infty}a_iz^i=\frac{(1-r)q}{1-(1-r)(1-q)}+$$

$$+z\frac{rq}{1-(1-r)(1-q)}+z\frac{r(1-q)(1-r+rz)^n[1-(1-q)^n]}{[1-(1-r)(1-q)][1-(1-q)^n(1-r+rz)^n]},$$

$$B(z)=\sum_{i=0}^{\infty}b_iz^i=\frac{1-(1-r)^n(1-r+rz)^n}{1-(1-r)(1-r+rz)}\frac{r(1-r+rz)}{1-(1-r)^n}+$$

$$+\frac{1-(1-r)^n(1-q)^n(1-r+rz)^n}{1-(1-r)(1-q)(1-r+rz)}\frac{r(1-q)(1-r+rz)[(1-r+rz)^n-1]}{[1-(1-r)^n][1-(1-q)^n(1-r+rz)^n]}.$$

For the stationary distribution we obtained

$$P(z)=\sum_{i=0}^{\infty}p_iz^i=p_0\frac{zA(z)-B(z)+\frac{rz}{(1-r)q}[A(z)-B(z)]}{z-B(z)},$$

where

$$p_0 = \frac{(1-r)q}{1-(1-r)(1-q)} - \frac{r(1-q)[1-(1-r)^n]}{(1-r)^{n-1}[1-(1-q)^n][1-(1-r)(1-q)]}.$$

The ergodicity condition was

$$\frac{r(1-q)}{1-(1-q)^n}\frac{1-(1-r)^n(1-q)^n}{1-(1-r)(1-q)} < (1-r)^n. \tag{3}$$

Now we shortly repeat some of Koba's results [1]. Let us denote by t_n the moment of entry of the n-th customer, then its service starts at the moment $t_n + T \cdot X_n$, where X_n is always a nonnegative integer. Let $Z_n = t_{n+1} - t_n$, S_n the service time of the n-th customer. Between X_n and X_{n+1} we have the following relation. Let $X_n = i$. If $(k-1)T < iT + S_n - Z_n \leq kT$, where $k \geq 1$ and integer, then $X_{n+1} = k$; if $iT + S_n - Z_n \leq 0$, then $X_{n+1} = 0$. So, X_n is a homogeneous Markov chain with transition probabilities p_{ik}, where

$$p_{ik} = P\{(k-i-1)T < S_n - Z_n \leq (k-i)T\}$$

if $k \geq 1$;

$$p_{i0} = P\{S_n - Z_n \leq -iT\}.$$

We introduce the notation

$$f_j = P\{(j-1)T < S_n - Z_n \leq jT\}, \tag{4}$$

$$p_{ik} = f_{k-i} \quad \text{if} \quad k \geq 1 \quad \text{and} \quad p_{i0} = \sum_{j=-\infty}^{-i} f_j = \hat{f}_i. \tag{5}$$

By using these probabilities the stationary distribution satisfies the system of equations

$$p_j = \sum_{i=0}^{\infty} p_i p_{ij}, \qquad j \geq 0, \tag{6}$$

$$\sum_{j=0}^{\infty} p_j = 1.$$

3. Results

By using Koba's results [1] our purpose is to find the transition probabilities and the solution of (6) for discrete and continuous cyclic-waiting systems and to derive the stability condition for them. For the stationary distribution of waiting time we will use the notation $P(z) = \sum_{j=0}^{\infty} p_j z^j$ (in the previous paragraph it was used for the queue length). We formulate our results in the following theorems.

Theorem 1. *Let us consider a queueing system in which the interarrival and service times are geometrically distributed (they are equal to i time units with probabilities $(1-r)^{i-1}r$ and $(1-q)^{i-1}q$, respectively); the service of a customer may be started upon arrival (if the server is free and there is no queue) or at moments differing from it by the multiples of cycle time T equal to n time units (in case of busy server or queue) according to the FCFS rule. Let us define an embedded Markov chain whose states correspond to the waiting time at the moments of arrivals of the customers. The matrix of transition probabilities has the form*

$$\begin{bmatrix} \sum_{j=-\infty}^{0} f_j & f_1 & f_2 & f_3 & f_4 & \cdots \\ \sum_{j=-\infty}^{-1} f_j & f_0 & f_1 & f_2 & f_3 & \cdots \\ \sum_{j=-\infty}^{-2} f_j & f_{-1} & f_0 & f_1 & f_2 & \cdots \\ \sum_{j=-\infty}^{-3} f_j & f_{-2} & f_{-1} & f_0 & f_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{7}$$

its elements are determined by (4)-(5). The generating function of ergodic distribution has the form

$$P(z) = \left[1 - \frac{r}{q}\frac{1-q}{(1-r)^n}\frac{1-(1-r)^n}{1-(1-q)^n}\right] \times \tag{8}$$

$$\times \frac{\dfrac{q}{1-(1-r)(1-q)}\left[1 - \dfrac{z[1-(1-r)^n]}{z-(1-r)^n}\right]}{1 - \dfrac{r(1-q)[1-(1-q)^n]}{1-(1-r)(1-q)}\dfrac{z}{1-(1-q)^n z} - \dfrac{q[1-(1-r)^n]}{1-(1-r)(1-q)}\dfrac{z}{z-(1-r)^n}},$$

the condition of existence of ergodic distribution is

$$\frac{r}{q}\frac{(1-q)}{(1-r)^n}\frac{1-(1-r)^n}{1-(1-q)^n}<1.$$

Theorem 2. *Let us consider a queueing system in which the arriving customers form a Poisson process with parameter λ, the service time distribution is exponential with parameter μ; the service of a customer may be started upon arrival (if the server is free and there is no queue) or at moments differing from it by the multiples of cycle time T (in case of busy server or queue) according to the FCFS rule. Let us define an embedded Markov chain whose states correspond to the waiting time at the moment of arrival of the customers. The matrix of transition probabilities has the form (7), its elements are determined by (4)-(5). The generating function of ergodic distribution has the form*

$$P(z)=\left[1-\frac{\lambda}{\mu}\frac{1-e^{-\lambda T}}{e^{-\lambda T}(1-e^{-\mu T})}\right]\times \tag{9}$$

$$\times\frac{\dfrac{\mu}{\lambda+\mu}-\dfrac{\mu(1-e^{-\lambda T})}{\lambda+\mu}\dfrac{z}{z-e^{-\lambda T}}}{1-\dfrac{\lambda(1-e^{-\mu T})}{\lambda+\mu}\dfrac{z}{1-ze^{-\mu T}}-\dfrac{\mu(1-e^{-\lambda T})}{\lambda+\mu}\dfrac{z}{z-e^{-\lambda T}}},$$

the condition of existence of ergodic distribution is the fulfilment of inequality

$$\frac{\lambda}{\mu}<\frac{e^{-\lambda T}(1-e^{-\mu T})}{1-e^{-\lambda T}}.$$

4. Proofs

In this part we find the ergodic distribution of waiting time both in discrete and continuous time cases, i.e. we prove the theorems.

4.1. The discrete time case

We determine the distribution of S_n-Z_n if both of them are geometrically distributed. The probability that $S_n-Z_n=j$ is equal to

$$\sum_{i=1}^{\infty}(1-r)^{i-1}r(1-q)^{i-1+j} = \frac{rq(1-q)^j}{1-(1-r)(1-q)}$$

if $j > 0$; and

$$\sum_{i=1}^{\infty}(1-q)^{i-1}q(1-r)^{i-1-j}r = \frac{rq(1-r)^{-j}}{1-(1-r)(1-q)}$$

if $j \le 0$. The transition probabilities are given by the formulas

$$f_j = \sum_{k=(j-1)n+1}^{jn} \frac{rq(1-q)^k}{1-(1-r)(1-q)} = \frac{r(1-q)[1-(1-q)^n]}{1-(1-r)(1-q)}(1-q)^{(j-1)n} \tag{10}$$

in case of positive jumps, and

$$f_{-j} = \sum_{k=jn}^{(j+1)n-1} \frac{rq(1-r)^k}{1-(1-r)(1-q)} = \frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}(1-r)^{jn} \tag{11}$$

for the case of nonpositive jumps. Furthermore, we have

$$p_{i0} = \sum_{k=-\infty}^{-i} f_k = \sum_{j=i}^{\infty} \frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}(1-r)^{jn} = \frac{q(1-r)^{in}}{1-(1-r)(1-q)} = \hat{f}_i. \tag{12}$$

By using the transition probabilities (4)-(5) the system of equations (6) has the form

$$\begin{aligned}
p_0 &= p_0\hat{f}_0 + p_1\hat{f}_1 + p_2\hat{f}_2 + p_3\hat{f}_3 + \dots \\
p_1 &= p_0 f_1 + p_1 f_0 + p_2 f_{-1} + p_3 f_{-2} + \dots \\
p_2 &= p_0 f_2 + p_1 f_1 + p_2 f_0 + p_3 f_{-1} + \dots \\
&\vdots
\end{aligned}$$

Multiplying the j-th equation by z^j and summing up from 0 to infinity for the generating function $P(z) = \sum_{j=0}^{\infty} p_j z^j$ we obtain

$$P(z) = P(z)F_+(z) + \sum_{j=1}^{\infty} p_j z^j \sum_{i=0}^{j-1} f_{-i} z^{-i} + \sum_{j=0}^{\infty} p_j \hat{f}_j, \tag{13}$$

where

$$F_+(z)=\sum_{i=1}^{\infty} f_i z^i .$$

By using the values (10)-(11)

$$F_+(z)=\sum_{i=1}^{\infty}\frac{r(1-q)[1-(1-q)^n]}{1-(1-r)(1-q)}(1-q)^{(i-1)n}z^i=$$

$$=\frac{r(1-q)[1-(1-q)^n]}{1-(1-r)(1-q)}\frac{z}{1-(1-q)^n z},$$

$$\sum_{i=0}^{j-1} f_{-i}z^{-i}=\sum_{i=0}^{j-1}\frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}\left(\frac{(1-r)^n}{z}\right)^i=$$

$$=\frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}\frac{1-\left(\frac{(1-r)^n}{z}\right)^j}{1-\frac{(1-r)^n}{z}},$$

$$\sum_{i=1}^{\infty}p_i\hat{f}_i=\sum_{i=0}^{\infty}p_i\frac{q(1-r)^{in}}{1-(1-r)(1-q)}=$$

$$=\frac{q}{1-(1-r)(1-q)}\sum_{i=0}^{\infty}p_i(1-r)^{in}=\frac{q}{1-(1-r)(1-q)}P\left((1-r)^n\right),$$

$$\sum_{j=1}^{\infty}p_j z^j\sum_{i=0}^{j-1} f_{-i}z^{-i}=\sum_{j=1}^{\infty}p_j z^j\frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}\frac{1-\left(\frac{(1-r)^n}{z}\right)^j}{1-\frac{(1-r)^n}{z}}=$$

$$=\frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}\frac{1}{1-\frac{(1-r)^n}{z}}\sum_{j=1}^{\infty}p_j[z^j-(1-r)^{nj}]=$$

$$=\frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}\frac{z}{z-(1-r)^n}\left[P(z)-P\left((1-r)^n\right)\right].$$

So, (13) may be written in the form

$$P(z)\left[1-\frac{r(1-q)[1-(1-q)^n]}{1-(1-r)(1-q)}\frac{z}{1-(1-q)^n z}-\right.$$

$$\left.-\frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}\frac{z}{z-(1-r)^n}\right]=$$

$$= P\left((1-r)^n\right)\left[\frac{q}{1-(1-r)(1-q)} - \frac{q[1-(1-r)^n]}{1-(1-r)(1-q)}\frac{z}{z-(1-r)^n}\right].$$

This expression contains the unknown value $P((1-r)^n)$, it can be found from the condition $P(1) = 1$. It is equal to

$$P((1-r)^n) = 1 - \frac{r}{q}\frac{1-q}{(1-r)^n}\frac{1-(1-r)^n}{1-(1-q)^n},$$

so, finally, the generating function takes the form (8). The chain is irreducible and aperiodic, in order to get the ergodicity condition we find p_0 from (13)

$$p_0 = \left[1 - \frac{r}{q}\frac{1-q}{(1-r)^n}\frac{1-(1-r)^n}{1-(1-q)^n}\right]\frac{q}{1-(1-r)(1-q)}.$$

It is positive if

$$\frac{r}{q}\frac{1-q}{(1-r)^n}\frac{1-(1-r)^n}{1-(1-q)^n} < 1,$$

this is equivalent to (3).

4.2. The continuous time case

Let Z denote the interarrival, S the service time, and

$$P\{Z < x\} = 1 - e^{-\lambda x} \qquad \text{and} \qquad P\{S < x\} = 1 - e^{-\mu x}.$$

Let us find the distribution of $S - Z$. If $x > 0$, then $S < Z + x$ and

$$P\{S - Z < x\} = \int_0^\infty [1 - e^{-\mu(x+y)}]\lambda e^{-\lambda y}dy = 1 - \frac{\lambda}{\lambda+\mu}e^{-\mu x}.$$

If $x < 0$, then from $S - Z < x$ follows $S - x < Z$, i.e.

$$P\{S - Z < x\} = \int_0^\infty e^{-\lambda(y-x)}\mu e^{-\mu y}dy = \frac{\mu}{\lambda+\mu}e^{\lambda x}.$$

For the distribution of $S - Z$ we have

$$F(x) = \begin{cases} \dfrac{\mu}{\lambda+\mu}e^{\lambda x} & \text{if } x \le 0, \\ 1 - \dfrac{\lambda}{\lambda+\mu}e^{-\mu x} & \text{if } x > 0. \end{cases}$$

We find the transition probabilities of the Markov chain: for $j > 0$

$$f_j = 1 - \frac{\lambda}{\lambda+\mu}e^{-\mu(j-1)T} - 1 + \frac{\lambda}{\lambda+\mu}e^{-\mu jT} = \frac{\lambda}{\lambda+\mu}(1-e^{-\mu T})e^{-\mu(j-1)T},$$

for the negative values ($j \geq 0$)

$$f_{-j} = \frac{\mu}{\lambda+\mu}e^{-\lambda jT} - \frac{\mu}{\lambda+\mu}e^{-\lambda(j+1)T} = \frac{\mu}{\lambda+\mu}(1-e^{-\lambda T})e^{-\lambda jT},$$

$$p_{i0} = \hat{f}_i = \sum_{j=-\infty}^{-i} f_j = \sum_{j=i}^{\infty} \frac{\mu}{\lambda+\mu}(1-e^{-\lambda T})e^{-\lambda jT} = \frac{\mu}{\lambda+\mu}e^{-\lambda iT}.$$

The matrix of transition probabilities has the form (7), by using it we come again to the generating function (13). In our case we have

$$F_+(z) = \sum_{i=1}^{\infty} f_i z^i = \frac{\lambda z}{\lambda+\mu}(1-e^{-\mu T})\sum_{i=1}^{\infty} e^{-\mu(i-1)T}z^{i-1} =$$

$$= \frac{\lambda(1-e^{-\mu T})}{\lambda+\mu}\frac{z}{1-ze^{-\mu T}},$$

$$\sum_{i=0}^{j-1} f_{-i}z^{-i} = \frac{\mu(1-e^{-\lambda T})}{\lambda+\mu}\sum_{i=0}^{j-1} e^{-\lambda iT}z^{-i} = \frac{\mu(1-e^{-\lambda T})}{\lambda+\mu}\frac{1-\left(\frac{e^{-\lambda T}}{z}\right)^j}{1-\frac{e^{-\lambda T}}{z}},$$

$$\sum_{i=0}^{\infty} p_i\hat{f}_i = \sum_{i=0}^{\infty} p_i\frac{\mu}{\lambda+\mu}e^{-\lambda iT} = \frac{\mu}{\lambda+\mu}P\left(e^{-\lambda T}\right).$$

By using these values we obtain

$$P(z) = P(z)F_+(z) + \sum_{j=1}^{\infty} p_j z^j \frac{\mu(1-e^{-\lambda T})}{\lambda+\mu}\frac{1-\left(\frac{e^{-\lambda T}}{z}\right)^j}{1-\frac{e^{-\lambda T}}{z}} + \frac{\mu}{\lambda+\mu}P\left(e^{-\lambda T}\right) =$$

$$= P(z)F_+(z) + \frac{\mu(1-e^{-\lambda T})}{\lambda+\mu}\frac{z}{z-e^{-\lambda T}}\left[P(z) - P\left(e^{-\lambda T}\right)\right] + \frac{\mu}{\lambda+\mu}P\left(e^{-\lambda T}\right),$$

or

$$P(z)\left[1 - F_+(z) - \frac{\mu(1-e^{-\lambda T})}{\lambda+\mu}\frac{z}{z-e^{-\lambda T}}\right] =$$

$$= P\left(e^{-\lambda T}\right)\left[\frac{\mu}{\lambda+\mu} - \frac{\mu(1-e^{-\lambda T})}{\lambda+\mu}\frac{z}{z-e^{-\lambda T}}\right].$$

In order to find the value $P\left(e^{-\lambda T}\right)$ we use the fact $P(1)=1$, from which

$$P\left(e^{-\lambda T}\right)=1-\frac{\lambda}{\mu}\frac{1-e^{-\lambda T}}{e^{-\lambda T}(1-e^{-\mu T})}.$$

For the generating function $P(z)$ we get the expression (9). From it the probability of zero waiting time

$$p_0=\left[1-\frac{\lambda}{\mu}\frac{1-e^{-\lambda T}}{e^{-\lambda T}(1-e^{-\mu T})}\right]\frac{\mu}{\lambda+\mu},$$

in order to have $p_0>0$

$$\frac{\lambda}{\mu}\frac{1-e^{-\lambda T}}{e^{-\lambda T}(1-e^{-\mu T})}<1$$

must be fulfilled, it leads to the ergodicity condition (2).

References

[1] **Коба Е.В.,** О системе обслуживания $GI/G/1$ с повторением заявок при обслуживании в порядке очереди, *Доповіді НАН України,* (6) (2000), 101-103.

[2] **Коба Е.В. и Михалевич К.В.,** Сравнение систем обслуживания типа $M/G/1$ с повторением при быстром возвращении с орбиты, *Queues: Flows, Systems, Networks. Proc. of Int. Conf. "Modern Math. Methods of Telecommunication Networks", 23-25 September, 2003, Gomel,* BGU, Minsk, 2003, 136-138.

[3] **Коваленко И.Н.,** Вероятность потери в системе обслуживания $M/G/m$ с T-повторением вызовов в режиме малой нагрузки, *Доповіді НАН України,* (5) (2002), 77-80.

[4] **Laevens K., Van Houdt B., Blondia C. and Bruneel H.,** On the sustainable load of fiber delay line buffers, *Electronic Letters,* **40** (2004), 137-138.

[5] **Lakatos L.,** On a simple continuous cyclic-waiting problem, *Annales Univ. Sci. Budapest. Sect. Comp.,* **14** (1994), 105-113.

[6] **Lakatos L.,** On a cyclic-waiting queueing system, *Theory of Stochastic Processes,* **2(18)** (1-2) (1996), 176-180.

[7] **Lakatos L.,** On a simple discrete cyclic-waiting queueing problem, *J. Math. Sci. (New York),* **92** (4) (1998), 4031-4034.

[8] **Lakatos L.,** A retrial system with time-limited tasks, *Theory of Stochastic Processes,* **8(24)** (3-4) (2002), 250-256.

[9] **Rogiest W., Laevens K., Fiems D. and Bruneel H.,** Analysis of a Lakatos-type queueing system with general service times, *Proc. of ORBEL 20. Quantitative Methods for Decision Making, Ghent, January 19-20, 2006,* 95-97.

(Received February 23, 2007)

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