CHOOSING APPROPRIATE DISTANCE MEASUREMENT IN DIGITAL IMAGE SEGMENTATION

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Dedicated to Professor Imre Kátai on his 65th birthday

Abstract. Finding certain objects in digital images is called image segmentation in digital image processing. Many segmentation techniques are based on the fact that objects usually consist of pixels having similar colour tints. To measure the differences between the colour values of the image pixels, traditionally classical distance functions (e.g. the Euclidean one) are considered. In this paper we show how we can take advantage of using less known distance functions in image processing applications. As a special family, we perform a detailed analysis for distance functions generated by neighbourhood sequences. These distance functions are integer valued ones, and thus their application is quite natural and descriptive for images, when the colour coordinates of the pixels are non-negative integers. Moreover, with taking advantage of the fact that neighbourhood sequences do not generate metrics in general, we can carry through new ideas in distance measurement that cannot be achieved by metrics. To show efficiency of our approach, we build it into such well-known image segmentation algorithms that are based on distance measurement, namely the definition of colour ranges, segmenting objects by region growing and classifying image pixels by the tools of cluster analysis. We also introduce some tools to help with finding the appropriate distance function to have optimal results.

1. Introduction

Extracting relevant data from multidimensional (e.g. colour) images are important operations with growing interest [19]. If the problem is to find objects in the images then these operations are known as segmentation methods in the literature of digital processing. A rich family of these procedures make use of the fact that objects are usually defined by a group of neighbouring image pixels having similar colour data. As a natural consequence, these algorithms need some kind of distance measurement to compare the colours of the pixels. Traditionally, most of these image segmentation techniques are based on classical (e.g. Euclidean) metrics. In some cases other metrics may provide better results [7], but very few suggestions can be found in the literature how to choose them. Especially, if the domain of the colour representation is some discrete set then integer valued distance functions are also worth considering. Beside the simplicity of handling digital distance functions they often have more illustrative behaviour than classic metrics. Another important point is that these segmentation techniques are based only on distance measurement and the satisfaction of the triangle inequality is not a natural requirement to hold. In other words such distance functions also may behave well that are actually not metrics.

The above observations has led us to the investigation of the applicability of a special family of digital distance functions in multidimensional image segmentation problems. Our approach is based on distance functions generated by neighbourhood sequences, as to every neighbourhood sequence a distance function can be assigned in natural way. The usefulness of special neighbourhood sequences in image processing applications was already noted in the very first fundamental papers of digital image processing (see [20]), and also in [13] for indexing and segmenting purposes. The main advantage of these types of distance functions is that they can be introduced in arbitrary finite dimension and also on different types of grid. Moreover, we can also take advantage the fact that these distance functions are not metrics in general.

We observe three classical image processing applications that are based on distance measurement, namely the definition of colour ranges, segmenting objects by region growing and classifying image pixels by the tools of cluster analysis. Some of these procedures use additional parameters and to help with adjusting them, we propose some tools, namely some histograms composed from the distance between the colours of the pixels. Using these tools we illustrate the behaviour of classical metrics and distance functions based on neighbourhood sequences, and how we can take advantage of using the latter family. Moreover, we show our proposed tools help with choosing parameters to obtain optimal results. As a very general colour representation in image processing, in our investigations we focus on the 3D Red-Green-Blue (RGB) domain.

The structure of the paper is as follows. In Section 2 we recall some concepts and results from the theory of neighbourhood sequences that we need in our further analysis. Section 3 explains the application of neighbourhood sequences in the RGB-domain for image segmentation purposes. In Section 4 we present concrete techniques for color image segmentation using different distance functions. Finally, in Section 5 we summarize our results and indicate some corresponding open problems.

2. Neighbourhood sequences

The classical digital (cityblock and chessboard) motions were introduced by Rosenfeld and Pfaltz [20] for $\mathbb{Z}^2$ (where $\mathbb{Z}^2$ denotes the integer number set). A cityblock step allows horizontal and vertical directions, while the chessboard step diagonal directions, as well. According to these two types of steps, the authors in [20] defined two distances. The d_4 or d_8 distance of two points is the number of steps needed to get from one of the points to the other, where only cityblock or chessboard steps are allowed, respectively. As a better approximation of the Euclidean distance, Rosenfeld and Pfaltz suggested to alternate the cityblock and chessboard steps, which led to the distance d_{oct}.

Later, Das et al. [5] introduced the concept of periodic neighbourhood sequences using arbitrary periodic mixture of the cityblock and chessboard steps, and generalised the theory to arbitrary dimension. They also derived a closed formula for calculating the distance between two points in arbitrary dimension determined by a periodic neighbourhood sequence. However, distance functions generated by neighbourhood sequences are not always metrics. In [5] Das et al. gave criteria to decide whether a neihbourhood sequence generates a metric or not. Based on the formula for calculating the distance, Das in [4] introduced a "natural" partial ordering relation on the set of periodic 2D-neighbourhood sequences. Later, Fazekas in [8] extended this result to neighbourhood sequences in 3D.

By dropping the periodicity condition, Fazekas et al. [9] extended the concept of neighbourhood sequences. Let us fix an arbitrary positive integer n for the whole paper. Let q and r be two points in $\mathbb{Z}^n$. The i-th coordinate of the point q is indicated by $\mathrm{Pr}_i(q)$. Let m be an integer with $1 \le m \le n$. The points q and r are m-neighbours, if the following two conditions hold:

- $|\mathrm{Pr}_i(q) - \mathrm{Pr}_i(r)| \le 1 \qquad (1 \le i \le n)$,
- $\sum_{i=1}^{n} |\mathrm{Pr}_i(q) - \mathrm{Pr}_i(r)| \le m$.

The sequence $A = \{A(i)\}_{i=l}^{\infty}$, where $A(i) \in \{1, ..., n\}$ for all $i \in \mathbb{N}$, is called an n-dimensional (shortly nD) neighbourhood sequence. If for some $l \in \mathbb{N}$, $A(i+$ $+l) = A(i)$ $(i \in \mathbb{N})$, then A is periodic with period l. In this case we briefly write $A = \{A(1)A(2)...A(l)\}$. For example, we write $\{12\}$ for the neighbourhood sequences $\{1, 2, 1, 2, 1, 2, ...\}$. A point sequence $q = q_0, q_1, ..., q_t = r$, where q_{i-1} and q_i are $A(i)$-neighbours in $\mathbb{Z}^n$ $(1 \le i \le t)$, is called an A-path from q to r of length t. The A-distance $d(q, r; A)$ of q and r is defined as the length of shortest A-path(s) between them. A closed formula for calculating the A-distance is given in [9] for any neighbourhood sequence A.

The former results on the "natural" ordering relation were extended in [9], as well. Namely, for two nD-neighbourhood sequences A and B. A is called "faster" than B if $d(q, r; A) \le d(q, r.B)$ for any $q, r \in \mathbb{Z}^n$. The authors in [9] gave criteria in arbitrary dimension to compare neighbourhood sequences with respect to this ordering. This "natural" ordering has some unpleasant structural properties (it fails to be a complete ordering) on the set of neighbourhood sequences . However in some applications (e.g. in those presented in this paper), it is useful to compare any two neighbourhood sequences, i.e. to decide which one spreads "faster". For this purpose a norm-like concept, called velocity, was introduced by A. Hajdu and L. Hajdu in [12] in a way to fit the relation "faster".

Distance functions generated by neighbourhood sequences are not metrics in general and the existence of this of this property can be checked by a criterion obtained by Nagy [16] extending the former results of Das et al. [5] for periodic sequences. As we show it later in our applications, non-metrical functions also provide nice results, thus it is not recommended to exclude them for analysis. Moreover, with these non-metrical distances we have a lot more distance functions to choose from to refine our results.

Since in many applications the so-called Minkowski distances

$$L_p(q, r) = \left(\sum_{i=1}^{n} |\mathrm{Pr}_i - \mathrm{Pr}_i(r)|^p \right)^{1/p}, \qquad p \ge 1,$$

$$L_\infty(q, r) = \max_{i=1}^{n} |\mathrm{Pr}_i(q) - \mathrm{Pr}_i(r)|$$

are considered, we show the relationship between the L_p metrics and the distance functions generated by neighbourhood sequences. Namely, we have

$$L_1(q,r) = d(q,r;\{1\}) \geq d(q,r;A) \geq d(q,r;\{n\}) = L_\infty(q,r)$$

for any nD-neighbourhood sequence A. L_p metrics also can be approximated by distance functions based on neighbourhood sequences, see e.g. [3,6,11,15] for the $p = 2$ case in 2D and 3D.

3. Using neighbourhood sequences in the RGB colour representation

Numerous colour image processing methods are based on the comparison of the colour of the pixels. The 24-bit RGB colour representation (that is the domain is between black= $(0,0,0)$ and white= $(255,255,255)$ with red= $(255,0,0)$, green= $(0,255,0)$, blue= $(0,0,255)$, yellow= $(255,255,0)$, magenta= $(255,0,255)$ and cyan= $(0,255,255)$ is a frequently applied domain. In this section, we will consider the behaviour of neighbourhood sequences in this 3D (RGB) colour representation in details. However, we note here that our procedures can be easily extended to arbitrary dimensional integer image representations, as well.

To calculate the distance generated by neighbourhood sequences, we recall a formula of Nagy [18] for the 3D space which has a simpler form than the one given for arbitrary dimension in [9]. For this purpose we need the concept of the limited neighbourhood sequences ([5,9]). The sequence $A^{(h)}$ with elements $A^{(h)}(i) = \min(A(i), h)$ is the h-dimensional limited sequence of A. The A-distance of the points $q, r \in \mathbb{Z}^3$ is given by $d(q,r;A) = \max\{v(1), d_2, d_3\}$, where

$$d_2 = \max\left\{ i \;\middle|\; v(1) + v(2) > \sum_{j=1}^{i-1} A^{(2)}(j) \right\},$$

and

$$d_3 = \max\left\{ i \;\middle|\; v(1) + v(2) + v(3) > \sum_{j=1}^{i-1} A(j) \right\},$$

and the values $v(j)$ $(j = 1,\ldots,3)$ correspond to the values $|\mathrm{Pr}_i(q) - \mathrm{Pr}_i(r)|$ $(i = 1,\ldots,3)$ ordered in a non-decreasing way, i.e. $v(1) = \max_i\{|\mathrm{Pr}(q) - \mathrm{Pr}_i(r)|\}$ and $v(3) = \min_i\{|\mathrm{Pr}(q) - \mathrm{Pr}_i(r)|\}$.

The criterion given for arbitrary dimension in [5,9] to check whether a neighbourhood sequence is "faster" than another one can be formulated as follows. In the 3D case let A and B 3D-neighbourhood sequences . Then $(q,r;A) \leq d(q,r;B)$ for all $q,r \in \mathbb{Z}^3$, if and only if $\sum_{j=1}^{i-1} A^{(2)}(j) \geq \sum_{j=1}^{i-1} B^{(2)}(j)$ and $\sum_{j=1}^{i-1} A(j) \geq \sum_{j=1}^{i-1} B(j)$. For example it can be easily checked that the periodic neighbourhood sequences $\{31\}$ and $\{2\}$ cannot be compared using this "natural" ordering relation.

As we noted earlier, neighbourhood sequences do not generate metrics in general. For the 3D case we formulate the general criterion of Nagy [16] given for arbitrary dimension. Namely, the 3D-neighbourhood sequence A generates a metric on $\mathbb{Z}^3$ if and only if $\sum_{i=k+1}^{k+j} A^{(h)}(i) \geq \sum_{i=1}^{j} A^{(h)}(i)$ for all $h,j,k \in \mathbb{N}$ with $1 \leq h \leq 3$. Roughly speaking, those neighbourhood sequences fail to generate metrics that become "slower" as e.g. $\{31\}$. (For such sequences the triangle inequality is not satisfied.)

Now, we illustrate the behaviour of neighbourhood sequences in measuring the distances between colours. First, let us consider the following three colours given by their RGB coordinates: $C_1(60,60,60)$ (dark grey), $C_2(180,180,180)$ (light grey), $C_3(240,30,90)$ (raspberry red). Calculating the distances of these colours with respect to the neighbourhood sequences $\{1\}$, $\{2\}$, and $\{3\}$, we get the distance values shown in Table 1.

	$A_1 = \{1\}$	$A_2 = \{2\}$	$A_3 = \{3\}$
$d(C_1,C_2)$	360	180	120
$d(C_1,C_3)$	240	180	180
$d(C_2,C_3)$	300	150	150

Table 1. The distances of colours depending on the neighbourhood sequence used

The entries of the table show that not only the distance values, but also the respective distances (i.e. the concepts "closer" and "farther") highly depend on the chosen neighbourhood sequences . More precisely, for the neighbourhood sequences $\{1\}$, $\{2\}$ and $\{3\}$, the colours (C_1,C_3), (C_2,C_3), and (C_1,C_2) are the closest, respectively.

The above used three neighbourhood sequences generate metrics. In the following example we show a distance function which does not meet the triangular inequality. Let $A_4 = \{3,2,1\}$, $C_4(30,30,30)$, $C_5(88,69,50)$ and $C_6(108,109,109)$. Then it is easy to calculate the distances:

$$d(C_4, C_5; A_4) = 58$$
$$d(C_5, C_6; A_4) = 59$$
$$d(C_4, C_6; A_4) = 118$$

As we can see the triangular inequality ($d(C_4, C_5; A_4) + d(C_5, C_6; A_4) \geq$ $\geq d(C_4, C_6; A_4)$) does not hold for these values.

In the next example we will use the non-periodic neighbourhood sequence $A_5 = \{1,1,1,2,2,2,2,2,3,3,\ldots\}$, with $A(i) = 3$ for all $i > 8$. Let us consider some A_5-distances in the RGB-cube:

$$d((0,0,0),(255,255,255); A_5) = 259$$
$$d((255,0,0),(0,0,255); A_5) = 257$$
$$d((0,255,0),(0,0,0); A_5) = 255$$
$$d((100,50,50),(200,250,100); A_5) = 200$$
$$d((75,175,124),(149,101,199); A_5) = 75$$

Moreover, A_5 generates a metric on $\mathbb{Z}^n$ and hence on the RGB-cube.

4. Applications in image segmentation based on distance functions

By involving distance functions generated by neighbourhood sequences , we can perform a more general distance measurement approach as usual in concrete colour image processing tasks. As we have a corresponding research on stone parts segmentation, the applicability of methods will be shown for images that contain stone particles captured by a microscope. The main challenge here is to detect special type of stones and to separate different stone parts. The whole segmentation procedure considers many image processing techniques among which the colour based segmentation is a basic approach. Now we present those segmentation methods that are found to be suitable in our research. Accordingly, we show the advances that can be achieved by considering new families of distance functions in defining colour ranges, performing segmentation by region growing and separating image segments by clustering.

Beside showing examples for the advantages of our approach, we also introduce some tools which nicely describe the behaviour of the distance measurement in the given image segmentation procedure. As the whole approach is based on the differences between colour values with respect to the chosen distance function, we found histograms composed from these values to be natural descriptors.

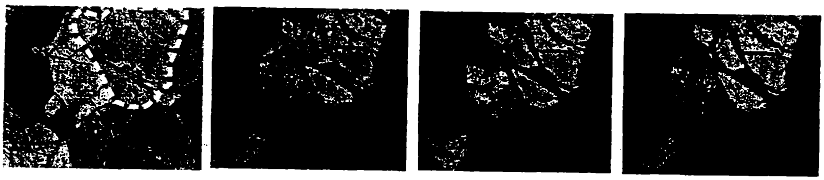
4.1. Colour ranges

Using the basic version of this procedure [10] we can find those pixels in the image whose colours are within a given threshold distance to an initially fixed seed colour. In the software realizations of this method usually one special function is fixed and the user should choose the seed colour and the threshold value without any guidelines.

Our purpose is to make it possible to change not only the seed colour and threshold value, but also the distance function to gain even more optimal results. Using a "faster" distance function with a lower threshold value and a "slower" distance function with a higher one, similar results can be obtained. Identical results rarely can be achieved in this way, since the occupied regions (spheres) of the distance functions geometrically differ, but the differences can be very small. However, using alternative distance measurement (e.g. distance functions based on neighbourhood sequences) we can gain additional possibilities beside just adjusting the speed of the distance function. Namely, neighbourhood sequences do not generate metrics in general, thus we can carry through such ideas that would have no sense in the case of classical metrics. One such idea in image segmentation is to start from a seed colour (object point) and grab many pixels with close colours rapidly as they are assumed to be object points. Then to avoid the inclusion of non-object points we want to decrease the inclusion of the number of pixels. To realize this idea a distance function is needed which becomes "slower". However, this property contradicts triangle inequality thus cannot be achieved by a metric.

To help with choosing a suitable distance function d, we use colour range histograms. After fixing a seed point x having seed colour c_x, the k-th column of this histogram represents the number of those image pixels y for which $k - \frac{1}{2} \leq d(c_x, c_y) < k + \frac{1}{2}$. Note that for integer valued distance functions we can use the simpler condition $d(c_x, c_y) = k$. Then a local minimum of this histogram is a natural choice for the threshold value, since local minima indicate those parts of the colour cube which colours are taken by a small number of image pixels. Especially, the first mode of the histogram (the closest colours to the seed) is possible to be the most important in applications. As the modality of the colour range histogram depends on the chosen distance function, a distance function is expected to perform better if the desired local minimum can be detected better in its corresponding histogram. For example, if the first local minimum has great importance, the first valley of the histogram should be as deep and wide as possible.

To illustrate the above ideas in choosing the suitable distance functions and corresponding threshold values we present an illustrative example. Let us consider the original image in Figure 1 (a), where the aim is to separate the stone part indicated by a dashed (manually defined) boundary. First we show the differences regarding to different distance functions, when one seed colour is fixed and the threshold value is adjusted accordingly to obtain approximately the same quality of results.



Original image (a) (b) (c)

Figure 1. Colour ranges from the seed colour = (221, 120, 162) based on different distance functions; (a) L_1 metric, threshold= 48, (b) L_∞ metric, threshold= 25, (c) distance function generated by $\{3333333333111\ldots\}$, threshold= 35.

It can be nicely observed that in this case the idea of choosing a distance function which becomes "slower" (namely the one generated by the neighbourhood sequence $\{3333333333111\ldots\}$ nicely worked, as it did not emerge that many non-object pixels as the metric L_1 or L_∞. To determine the suitable threshold values we used the corresponding colour range histograms shown in Figure 2.

Note that the histogram on Figure 2 (c) has the best behaviour when the focus is on the modality of the first mode. This behaviour has led to the better quality of result shown in Figure 1 (c).

The colours of objects are usually not homogeneous. A usual reason is that illumination causes many colour shades which can be also observed in our images captured by a microscope. In this case it is a natural improvement to consider more seed colours to describe the objects. Using this technique, the threshold values obviously should be decreased and less non-object points will be involved. Colour range histograms can be used again to find optimal distan-

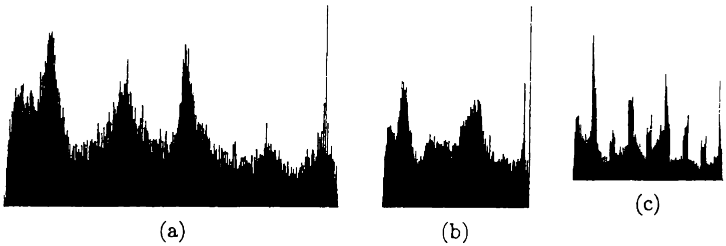


Figure 2. Colour range histograms accoprding to different distance functions: (a) L_1 metric, (b) L_∞ metric, (c) distance function generated by $\{3333333333111...\}$

ce functions with respect to each of the seed colours, separately. Our following example illustrates the advantages of using more seed colours with smaller threshold values. In Figure 3 the results shown in Figure 1 (a)-(c) are improved by considering more seed colours.

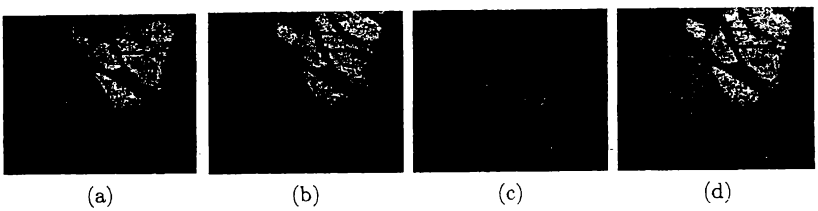


Figure 3. Colour ranges using three seed colours: (a) seed colour $= (223, 121, 158)$, L_∞ metric, threshold$= 20$. (b) seed colour $= (222, 129, 124)$, L_1 metric threshold$= 40$, (c) seed colour $= (228, 134, 135)$, L_∞ metric, threshold$= 10$ (d) union of (a), (b), (c).

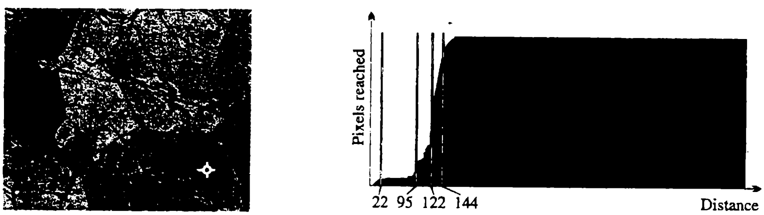
4.2. Region based segmentation

It is a very usual requirement in image segmentation methods that the resulted segments should be connected, where connectedness is defined based on the well-known 4- or 8-neighboring relations in 2D (which are equivalent, in the present terminology, to the 1- and 2-neighboring relations, respectively). A digital set is 4-connected (resp. 8-connected) if any two of its points can be reached by such a sequence of its points where the consecutive elements in the sequence are 4-neighbours (resp. 8-neighbours).

To preserve connectivity we insert our approach for colour ranges into a region growing algorithm. Region growing techniques are classical procedures (see [10, 21]) for finding connected segments that consist of pixels with similar colours. Starting from a seed point (inner point of the region) and fixing a threshold value we merge those neighbours of the seed point whose colours are within a distance to the colour of the seed point. Then we continue with merging those pixels that are neighbours previously merged ones and also have colours within the given threshold to the seed colour. The procedure stops, when no more pixels can be merged in this way.

It is clear that a natural extension of our colour range approach leads to a general region growing method. Namely, we achieve region growing with requiring spatial connectivity to the seed point beside considering colour range information. In this way we can also take advantage of the use of our alternative colour distance measurement also for region growing. Colour range histograms can be extended naturally for region growing purposes. Let a distance function d, a seed point x with colour c_x, and a threshold value T be fixed. Then the k-th column of the region growing histogram represents the number of those pixels y for which $k - \frac{1}{2} \leq d(c_x, c_y) < k + \frac{1}{2}$ and there exists a 4-path between x and y such that for any element z of the path $d(c_x, c_y) \leq T$. In Figure 4 we show an example for region growing histogram.

Note that the shape of the histogram is like a cumulative one, and its jumps indicate those distance values, where remarkable number of new pixels are merged. That is the suggested choices of thresholds are the distance values corresponding to the very beginning of the constant sections of the histogram, as it is shown in Figure 4 (b). The regions occupied by the algorithm when reaching the indicated values can be observed in Figure 5.



(a) (b)

Figure 4. Region growing of stone parts; (a) original image with seed point having seed colour = (85, 145, 59) indicated, (b) region growing histogram of the metric L_∞.

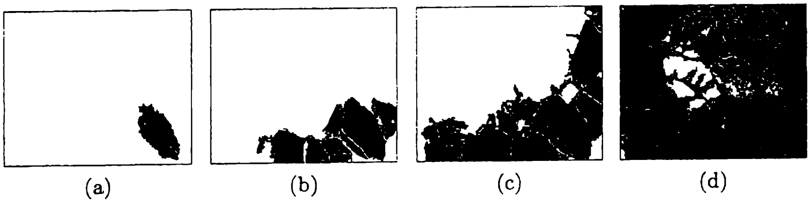


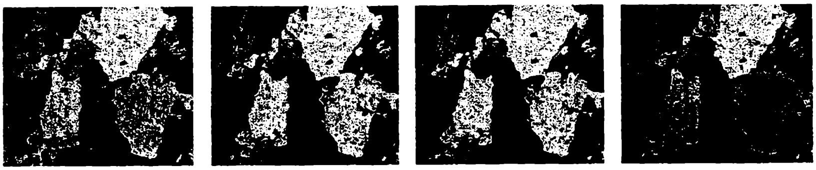
Figure 5. Occupied regions of region growing according to the threshold values (a) 22, (b) 95 (c) 122, (d) 144.

We can see that after occupying the first (desired) stone part a the distance 22, the next relevant change in region growing occurred only at the distance 95, when this amount of colour change let the algorithm grow over the crack around the seed stone part. The huge difference between Figure 5 (c) and (d) shows that in spite of the relatively small distance increment, the region growing histogram rapidly increases for larger values. This phenomenon easily can be explained by noting that after emerging a segment S having large colour distance from the seed, those segments which are connected with S and have closer distance values to the seed are immediately emerged.

The proposed region growing procedure naturally can be improved by using the optimisation techniques (more seed colours, modality analysis to choose appropriate distance functions, etc.) discussed at the colour range approach.

4.3. Classifying colours with cluster analysis

In such segmentation tasks if no a priori information or user interaction are available, automatic methods are needed. In such situations the widely applied methods of cluster analysis can be used. A huge number of clustering approaches are known, and basically all of them are based on some distance measurement. Thus the question also arises here, whether the application of the other distance functions than the classical one may lead to better results. As alternative possibilities, we used digital distance functions generated by neighbourhood sequences again, and achieved nice results. To compare the performance of different distance functions first we considered hierarchical clustering [1] with moving centroids that were rounded when integer valued distance functions were applied. To initialize this clustering procedure we put every colour in the image into separate clusters. Then at every further step we merge those two clusters which are closest to each other with respect to the chosen distance function. The algorithm stops when an initially fixed number of cluster is reached or when the distance of the closest two clusters is larger than an initially fixed threshold value. Since hierarchical clustering does not need any pre-processing or additional arguments we considered it as the most objective approach to compare the efficiency of different distance functions. In Figure 6 we show the results of hierarchical clustering obtained by using various distance functions.



(a) (b) (c) (d)

Figure 6. Results of hierarchical clustering into six clusters using different distance functions; (a) original image, (b) L_1 metric, (c) L_∞ metric, (d) distance functions generated by $\{123\}$.

It is always a very sophisticated problem in image processing to decide which method provides the "best" result. The reason is that the decision highly depends on the subjective impressions of the human receiver. Hence it is a usual practice to ask a group of people for taking sides in such questions. Nevertheless, these surveys are rather expensive and time consuming, and thus many error functions were composed to make these decision based on quantitative measures. To evaluate the results of clustering methods such a measure is the one proposed by Levine and Nazif [14]. In our investigations we used this uniformity measure with a slight modification and calculated

$$E = \frac{\sum_{i=1}^{L} \sigma_i^2}{L\sigma^2},$$

where L is the number of final clusters, σ_i^2 is the i-th within-class variance, while σ is the between-class variance. Note that $0 \leq E$, and the smaller E is the better, the corresponding result can be assumed. Especially, we have $E = 0$ if we put every group of pixels with the same colour in separate clusters. In Table 2 we present our results on clustering Figure 6 (a) into six clusters with different distance functions.

E	L_1	L_∞	{12}	{123}	{11113333}	{31}
Hierarchical	0,03432	0,03427	0,02362	0,02631	0,03426	0,07480
K-means	0,02862	0,02862	0,02555	0,02614	0,02610	0,02612

Table 2. Levine and Nazif error after classifying into six clusters using different distance functions and clustering methods

The quantitative analysis in Table 2 nicely shows that neighbourhood sequences are worth taking in consideration in clustering procedures, as well. In our special investigation on segmenting stone parts, hierarchical clustering does not seem to be a good choice in general, since no hierarchy can be observed in these types of images. Hence we observed another type of, namely the K-means, clustering method (see [1]). The classic version of this algorithm determines K initial cluster centroids and then puts every element into the cluster having the closest centroid. Table 2 contains our experimental results according to the Levine and Nazif uniformity measure for the same task as in the case of hierarchical clustering. From the table we can see that the K-means clustering procedure yields more reliable results (at least quantitatively) than the hierarchical one. Moreover, these results show that distance functions based on neighbourhood sequences can be nicely applied here, as well.

5. Conclusions

In this paper we showed how the special family of distance functions generated by neighbourhood sequences can be applied successfully in colour image segmentation beside classical metrics. It was also explained how the choice of different distance functions may improve the results. These methods are based on the suitable choice of distance functions and threshold values to define ranges around fixed seed colours. Our approach was given for the RGB colour representation, but they can be extended to arbitrary dimension [5,9,16] and also to other types of girds [17]. By the help of the relation introduced in [5,9] or the velocity value in [12], we can choose faster or slower distance functions or we can even ignore metrical properties [9,16]. As an application area, we presented stone parts segmentation, where the main task is to extract the different types of stones from an image captured by microscope.

In this topic there are still some unsolved problems like finding the suitable seed colours, threshold values and neighbourhood sequences to achieve optimal results. The precise mathematical solution of these problems is quite sophisticated even for the reason that the goodness of the result may be subjective decision. We defined some theoretical tools (histograms) that help with finding the suitable parameters, as the modes of the histograms will represent segment classes in the image.

Acknowledgments. The authors are grateful to János Farkas for making stone images available for the research, and to Tamás Tóth for his help with implementing algorithms.

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(Received January 28, 2004)

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