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DEVELOPMENT OF THE METHODS OF COMPUTER MODELLING FOR THE ELECTROMAGNETIC SOUNDING IN STRATIFIED MEDIA

V.I. Dmitriev (Moscow, Russia)
R.H. Farzan (Győr, Hungary)

Dedicated to Professor Imre Kátai on his 65-th birthday

Abstract. The integral equation method is frequently used for the solution of problems following from an electromagnetic sounding. Important element of using the integral equation method is a denoting of the Green tensor function. In the paper the algorithm for calculating the Green tensor is developed for the stratified space.

For the calculation of the hypersingular integrals in the Green tensor a formula is obtained for the principle value of the singular integral in a parallelepipedon.

1. Introduction

In our days among the different geophysical methods of the ore exploration the electromagnetic soundig method plays an important part. An electromagnetic field which can be measured on the surface of the Earth depends from the distribution of the electric conductivity inside a medium. A measured electromagnetic field gives information about the structure of upper layers of the Earth and specifically about the presence of inhomogeneities who give marks for ores. The efficiency of the electromagnetic sounding used depends on the elaboration of the fast methods for computer modelling of the electromagnetic fields in an inhomogeneous conductive media.

These scientific researches play an important role in the scientific cooperation of the Moscow's and Budapest's universities. The initiators of this cooperations were Prof. I.Kátai (from Hungarian side) and Prof. A.N.Tikhonov (from Russian side) (see [10]).

In our age special systems exist for the mathematical modelling of the propagation of electromagnetic fields in stratified media with local inhomogeneities, in those the electric conductivity is an arbitrary function. In this case a basic model is a stratified medium (with electric conductivity $\sigma(z)$ depending only on depth) which contains a local inhomogeneous domain V with an arbitrary electric conductivity $\sigma(x, y, z)$.

For this problem the integral equation method is frequently used ([5, 4, 9]). For reducing this problem to the solution of an integral equation in the local domain V we need know the Green tensor for the vector Maxwell equation in the stratified medium. Therefore an efficient algorithm is needed, which is adapted to using in the integral equation method, where values of the Green tensor can be computed fastly in many number of points of the domain V. Moreover, the singular part of the Green function must be separated analytically, because the integral equation, which is equivalent to a boundary-value problem for the Maxwell equations, is hypersingular (i.e. contains a non-integrable singularity) ([7]). Therefore beyond numerical solving of the integral equation a singularity of a kernel of the integral equation must be integrated analytically.

2. Formulation of the problem

The propagation of the electromagnetic waves is described by a system of Maxwell equations. The Maxwell equations formulated in frequency variables in the three-dimensional Euclidean space $\Re^3$ are ([2])

$$\text{rot}\mathbf{H} = -i\omega\mu_0\mathbf{E} + \mathbf{j}, \quad \text{rot}\mathbf{E} = i\omega\mu_0\mathbf{H}, \tag{1}$$

where $\mathbf{E}$, $\mathbf{H}$ are the vectors of the electric and magnetic fields, correspondingly, ω is the frequency. The medium is assumed to be stratified, magnetically homogeneous, i.e. a permeability is constant and equals to its value in vacuum: $\mu = \mu_0$, the conductivity $\sigma(z)$ is a piecewise continuous function of the depth only:

$$\begin{aligned} \sigma(z) =& \sigma_m, \quad z_{m-1} < z < z_m, \quad m = 1, 2, \dots N-1; \\ \sigma(z) =& \sigma_0 = const, \ z < z_0 = 0; \quad \sigma(z) = \sigma_N = const, \ z_{N-1} < z. \end{aligned} \tag{2}$$

For an arbitrary source function the vectors of electric and magnetic fields can be written in the integral form with the fundamental solutions, which are Green tensor functions of electric $\mathcal{E}(\mathbf{R},\mathbf{R}')$ and magnetic $\mathcal{H}(\mathbf{R},\mathbf{R}')$ types:

$$\mathbf{E}(\mathbf{R}) = \int_V \mathcal{E}(\mathbf{R},\mathbf{R}')\mathbf{j}(\mathbf{R}')d\mathbf{R}', \quad \mathbf{R}=(x,y,z), \quad \mathbf{R}'=(x',y',z'); \tag{3}$$

$$\mathbf{H}(\mathbf{R}) = \int_V \mathcal{H}(\mathbf{R},\mathbf{R}')\mathbf{j}(\mathbf{R}')d\mathbf{R}', \tag{4}$$

where V is a support of $\mathbf{j}$.

The Green tensor function is an analogue of the Green function for the case of the vector differential equation. $\mathcal{E}$ and $\mathcal{H}$ satisfy the equations

$$\mathrm{rot}\mathcal{H} = \sigma(z)\mathcal{E} + \mathcal{D}, \quad \mathrm{rot}\mathcal{E} = i\omega\mu_0\mathcal{H}; \tag{5}$$

where

$$\mathcal{D} = \delta(\mathbf{R}-\mathbf{R}')\mathcal{I}. \tag{6}$$

is the unit tensor, $\delta(\mathbf{R}-\mathbf{R}') = \delta(x-x')\delta(y-y')\delta(z-z')$ the three-dimensional scalar Dirac delta-function.

The Green tensors $\mathcal{E},\mathcal{H}$ can be represented by using the tensor-potential function $\mathcal{A}(\mathbf{R},\mathbf{R}')$ (see [5, 7]) as

$$\mathcal{H}(\mathbf{R},\mathbf{R}') = \frac{1}{i\omega\mu_0}\mathrm{rot}\mathcal{A}, \tag{7}$$

$$\mathcal{E}(\mathbf{R},\mathbf{R}') = \mathcal{A} + \nabla\left(\frac{1}{k^2}\mathrm{div}\mathcal{A}\right), \quad k^2 = i\omega\mu_0\sigma. \tag{8}$$

The tensor $\mathcal{A}$ satisfies the equation

$$\Delta\mathcal{A} + k^2\mathcal{A} + \mathcal{Q} = -\, i\omega\mu_0\mathcal{D}, \quad \mathcal{Q} = k^2\nabla\left(\frac{1}{k^2}\mathrm{div}\mathcal{A}\right) - \nabla\mathrm{div}\mathcal{A}, \tag{9}$$

where for the differential operators a variable is $\mathbf{R}$.

If k^2 is the function of z only we obtain for tensors $\mathcal{Q}$ and $\mathcal{A}$:

$$\mathcal{Q} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & & 0 \\ Q_z^x & Q_z^y & Q_z^z \end{pmatrix}; \tag{10}$$

and

$$\mathcal{A} = \begin{pmatrix} G_1(\mathbf{R},\mathbf{R}') & 0 & 0 \\ 0 & G_1(\mathbf{R},\mathbf{R}') & 0 \\ \dfrac{\partial g(\mathbf{R},\mathbf{R}')}{\partial x} & \dfrac{\partial g(\mathbf{R},\mathbf{R}')}{\partial y} & G_2(\mathbf{R},\mathbf{R}') \end{pmatrix}; \tag{11}$$

where the scalar functions $G_1,\ G_2$ and g satisfy the following equations:

$$\begin{gathered} \Delta G_1 + k^2 G_1 = -i\omega\mu_0\delta(\mathbf{R}-\mathbf{R}'), \\ k^2 \mathrm{div}\left(\frac{1}{k^2}\nabla G_2\right) + k^2 G_2 = -i\omega\mu_0\delta(\mathbf{R}-\mathbf{R}'), \\ k^2 \mathrm{div}\left(\frac{1}{k^2}\nabla g\right) + k^2 g = -k^2\frac{\partial}{\partial z}\left(\frac{1}{k^2}\right)G_1. \end{gathered} \tag{12}$$

On the surfaces $z = z_i,\ i = 0, 1, \ldots N-1$ (see (2)) the coefficients $\sigma(z)$ and $k(z)$ are discontinuous, and from the general conditions, that in the surfaces of discontinuity the tangential components of the electric and magnetic fields are continuous, we have for G_1, G_2 and g the conjugate conditions in the form

$$G_1,\ G_2,\ g,\ \frac{\partial G_1}{\partial z},\ \frac{1}{\sigma}\frac{\partial G_2}{\partial z},\ \frac{1}{\sigma}\left(\frac{\partial g}{\partial z} + G_1\right) \quad \text{are continuous.} \tag{13}$$

Now let us suppose that $\sigma(z)$ is a piecewise constant function, i.e. in (2) $\sigma_m = const,\ m = 1, 2, \ldots N-1$. In this case equations (12) are reduced to the following forms

$$\begin{gathered} \Delta G_1 + k^2(z) G_1 = -i\omega\mu_0\delta(\mathbf{R}-\mathbf{R}'), \\ \Delta G_2 + k^2(z) G_2 = -i\omega\mu_0\delta(\mathbf{R}-\mathbf{R}'), \\ \Delta g + k^2 g = 0, \end{gathered} \tag{14}$$

and the conjugate conditions

$$[G_1]_{z_{m-1}} = 0, \qquad \left[\frac{\partial G_1}{\partial z}\right]_{z_{m-1}} = 0,$$

(15) $$[G_2]_{z_{m-1}} = 0, \qquad \left[\frac{1}{\sigma}\frac{\partial G_2}{\partial z}\right]_{z_{m-1}} = 0,$$

$$[g]_{z_{m-1}} = 0, \qquad \left[\frac{1}{\sigma}\frac{\partial g}{\partial z}\right]_{z_{m-1}} = - \left[\frac{1}{\sigma}\right]_{z_{m-1}} G_1,$$

where for $\forall\phi \;\; [\phi]_{z_{m-1}} = \phi(z_{m-1}+0) - \phi(z_{m-1}-0),\; m = 1,\dots,N-1.$

At infinity the radiation conditions are satisfied.

3. Numerical algorithms for the Green tensor

Let us introduce the cylindrical co-ordinates

$$\{r,\theta,z\}, \quad r = \sqrt{(x-x_0)^2 + (y-y_0)^2}.$$

From the equations (14) and conditions (15) one can see that the functions G_1, G_2, g are independent of θ. Therefore it is useful to apply some integral transformations. We use the Hankel-transformation (see [5]).

Let the Bessel-operator

(16) $$I_0(f) = \int_0^\infty J_0(tr) f(z,t) t dt,$$

where J_0 is zero-order Bessel function of the first kind.

Let us define the representations

(17) $$\begin{aligned} G_1(r,z,z') &= \frac{i\omega\mu_0}{4\pi} \cdot I_0(V_1), \\ G_2(r,z,z') &= \frac{i\omega\mu_0}{4\pi} \cdot I_0(V_2), \\ g(r,z,z') &= \frac{i\omega\mu_0}{4\pi} \cdot I_0(v). \end{aligned}$$

After that for $V_1,\ V_2,\ v$ we obtain the boundary value problem for ordinary differential equations

(18) $$\frac{d^2 V_1}{dz^2} - \eta^2(z) V_1 = -2\delta(z-z'),$$

$$[V_1]_{z_{m-1}} = 0, \quad \left[\frac{dV_1}{dz}\right]_{z_{m-1}} = 0, \quad z_{m-1} \neq z', \quad m = 1, \dots, N-1;$$

(19)
$$\frac{d^2V_2}{dz^2} - \eta^2(z)V_2 = -2\delta(z - z'),$$

$$[V_2]_{z_{m-1}} = 0, \quad \left[\frac{1}{\sigma}\frac{dV_2}{dz}\right]_{z_{m-1}} = 0, \quad z_{m-1} \neq z', \quad m = 1, \dots, N-1;$$

(20)
$$\frac{d^2v}{dz^2} - \eta^2(z)v = 0,$$

$$[v]_{z_{m-1}} = 0, \quad \left[\frac{1}{\sigma}\frac{dv}{dz}\right]_{z_{m-1}} = -\left[\frac{1}{\sigma}\right]_{z_{m-1}} V_1, \quad z_{m-1} \neq z', \quad m = 1, \dots, N-1,$$

where $\eta^2 = t^2 - k^2, \quad \mathrm{Re}(\eta) > 0$.

The problem (20) can be reduced to a simpler one if we define the new function

(21)
$$W(z,t) = t^2 v(z,t) + \frac{dV_1}{dz}.$$

This function satisfies the problem

(22)
$$\frac{d^2W}{dz^2} - \eta^2(z)W = -2\delta(z - z'),$$

$$[W]_{z_{m-1}} = 0, \quad \left[\frac{1}{\sigma}\frac{dW}{dz}\right]_{z_{m-1}} = 0, \quad z_{m-1} \neq z', \quad m = 1, \dots, N-1.$$

Now since (17) and (21) we have

(23)
$$g(r,z,z') = \frac{i\omega\mu_0}{4\pi}\left(I_0\left(\frac{W}{t^2}\right) - I_0\left(\frac{1}{t^2}\frac{dV_1}{dz}\right)\right).$$

The problems (18), (19), (22) can be reduced to a single problem for the common fundamental function $U_a^\alpha(z,t)$. For this function we can write ([2])

$$\frac{d^2U_a^\alpha}{dz^2} - \eta^2(z)U_a^\alpha = 0, \quad z \in (-\infty, z')\bigcup(z', \infty),$$

$$[U_a^\alpha]_{z_{m-1}} = 0, \quad \left[\frac{1}{a(z)}\frac{dU_a^\alpha}{dz}\right]_{z_{m-1}} = 0,\ z_{m-1} \neq z',\ m = 1, \dots, N-1,$$

$$[U_a^\alpha]_{z'} = -2a, \quad \left[\frac{d^2U_a^\alpha}{dz^2}\right]_{z'} = -2(1-\alpha), \tag{24}$$

$$U_a^\alpha \to 0 \quad \text{if } |z| \to \infty,$$

where $\alpha = 0, 1;\ a = 1, \sigma$. Now using the fundamental function U_a^α we get

$$V_1 = U_1^0; \quad V_2 = U_\sigma^0; \quad W = U_\sigma^1, \tag{25}$$

and, because of (17) and (23),

$$G_1 = \frac{i\omega\mu_0}{4\pi}\ I_0(U_1^0); \quad G_2 = \frac{i\omega\mu_0}{4\pi}\ I_0(U_\sigma^0); \tag{26}$$

$$g = \frac{i\omega\mu_0}{4\pi}\left(I_0\left(\frac{U_\sigma^1}{t^2}\right) - I_0\left(\frac{1}{t^2}\frac{dU_1^0}{dz}\right)\right).$$

After that for the components of the tensor $\mathcal{E}$ we obtain

$$\begin{aligned}
E_x^x &= \frac{i\omega\mu_0}{4\pi} I_0(U_1^0) + \frac{1}{4\pi\sigma(z)}\frac{\partial^2}{\partial x^2} I_0\left(\frac{1}{t^2}\left(k^2U_1^0 + \frac{dU_\sigma^1}{dz}\right)\right);\\
E_y^x &= E_x^y = \frac{1}{4\pi\sigma(z)}\frac{\partial^2}{\partial x \partial y} I_0\left(\frac{1}{t^2}\left(k^2U_1^0 + \frac{dU_\sigma^1}{dz}\right)\right);\\
E_y^y &= \frac{i\omega\mu_0}{4\pi} I_0(U_1^0) + \frac{1}{4\pi\sigma(z)}\frac{\partial^2}{\partial y^2} I_0\left(\frac{1}{t^2}\left(k^2U_1^0 + \frac{dU_\sigma^1}{dz}\right)\right);\\
E_z^x &= \frac{i\omega\mu_0}{4\pi}\frac{\partial}{\partial x} I_0(U_\sigma^1); \quad E_z^y = \frac{i\omega\mu_0}{4\pi}\frac{\partial}{\partial y} I_0(U_\sigma^1);
\end{aligned} \tag{27}$$

$$E_x^z = \frac{i\omega\mu_0}{4\pi}\frac{\partial}{\partial x} I_0\left(\frac{\partial U_\sigma^0}{\partial z}\right);\ E_y^z = \frac{i\omega\mu_0}{4\pi}\frac{\partial}{\partial y} I_0\left(\frac{\partial U_\sigma^0}{\partial z}\right);\ E_z^z = \frac{i\omega\mu_0}{4\pi} I_0(t^2U_\sigma^0).$$

For the derivatives of the Bessel operator $I_0(\phi)$ we can write

$$\begin{aligned}
&\frac{\partial}{\partial x} I_0(\phi) = -\frac{x-x'}{r} I_1(t\phi); \quad \frac{\partial}{\partial y} I_0(\phi) = -\frac{y-y'}{r} I_1(t\phi);\\
&\frac{\partial^2}{\partial x^2} I_0(\phi) = -\frac{(x-x')^2}{r^2} I_0(t^2\phi) + \frac{(x-x')^2 - (y-y')^2}{r^3} I_1(t\phi);\\
&\frac{\partial^2}{\partial y^2} I_0(\phi) = -\frac{(y-y')^2}{r^2} I_0(t^2\phi) + \frac{(y-y')^2 - (x-x')^2}{r^3} I_1(t\phi);\\
&\frac{\partial^2}{\partial x \partial y} I_0(\phi) = -\frac{(x-x')(y-y')}{r}\left(I_0(t^2\phi) - \frac{2}{r} I_1(t\phi)\right);
\end{aligned} \tag{28}$$

where operator

$$I_1(t\phi) = \int_0^\infty J_1(tr)\phi(t)t^2dt. \tag{29}$$

So the calculating problem for the electric Green tensor $\mathcal{E}$ have been reduced to solving the boundary-value problem for the ordinary differential equation (24) and integrating the Bessel integrals with fundamental function U_a^α.

Note that in [1] the algorithm is given for computing the Bessel integrals.

For the components of the magnetic Green tensor $\mathcal{H}$ we have from (7) and (11)

$$\begin{aligned}
&H_x^x = \frac{1}{i\omega\mu_0}\frac{\partial^2 g}{\partial x\partial y};\ H_y^x = \frac{1}{i\omega\mu_0}\left(\frac{\partial^2 g}{\partial y^2} - \frac{\partial G_1}{\partial z}\right);\ H_z^x = \frac{1}{i\omega\mu_0}\frac{\partial G_1}{\partial y};\\
&H_x^y = \frac{1}{i\omega\mu_0}\left(\frac{\partial G_1}{\partial z} - \frac{\partial^2 g}{\partial x^2}\right);\ H_y^y = -\frac{1}{i\omega\mu_0}\frac{\partial^2 g}{\partial x\partial y};\ H_z^y = \frac{1}{i\omega\mu_0}\frac{\partial G_1}{\partial x};\\
&H_x^z = \frac{1}{i\omega\mu_0}\frac{\partial G_1}{\partial y};\ H_y^z = \frac{1}{i\omega\mu_0}\frac{\partial G_1}{\partial x};\ H_z^z = 0.
\end{aligned} \tag{30}$$

The functions G_1, G_2, g can be computed from (26) as above.

4. Analysis of the singularity of the Green tensor

For an analysis of the singularitiy of the electric Green tensor in stratified media it is very important (and fortunational) that the singularity in any space is the same as it is in the homogeneous space. In the homogeneous space the Green function can be written in analytical form (see [5], [9]).

For the homogeneous medium

$$U_{10}^0 = U_{\sigma 0}^0 = \frac{1}{\eta_p}e^{-\eta_p|z-z'|}; \quad U_{\sigma 0}^1 = \frac{dU_{\sigma 0}^0}{dz}; \tag{31}$$

$$\frac{dU_{\sigma 0}^1}{dz} = \eta_p e^{-\eta_p|z-z'|}; \quad \frac{1}{t^2}\left(k_p^2 U_{10}^0 + \frac{dU_{\sigma 0}^1}{dz}\right) = U_{10}^0 = \frac{1}{\eta_p}e^{-\eta_p|z-z'|},$$

where $\eta_p = const$ is the value of η in the pole $\mathbf{R}'$.

Let us denote

$$A_0(\mathbf{R},\mathbf{R}') = I_0\left(\frac{1}{\eta_p}e^{-\eta_p|z-z'|}\right) = \int\limits_0^\infty J_0(tr)e^{-\eta_p|z-z'|}\frac{tdt}{\eta_p} = \frac{e^{ik_pR}}{R}, \tag{32}$$

where $R = \sqrt{r^2+(z-z')^2}$. Note, that the last equality in (32) is the well-known Sommerfeld formula. After that we can write the singular part of $\mathcal{E}$ as

$$\mathcal{E}^0 = \frac{i\omega\mu_0}{4\pi}A_0(\mathbf{R},\mathbf{R}')\mathcal{I} + \frac{1}{4\pi\sigma_p}(\nabla\cdot\nabla^T)A_0(\mathbf{R},\mathbf{R}'), \tag{33}$$

where the matrix differential operator

$$(\nabla\cdot\nabla^T) = \begin{pmatrix} \dfrac{\partial^2}{\partial x^2} & \dfrac{\partial^2}{\partial x\partial y} & \dfrac{\partial^2}{\partial x\partial z} \\ \dfrac{\partial^2}{\partial x\partial y} & \dfrac{\partial^2}{\partial y^2} & \dfrac{\partial^2}{\partial y\partial z} \\ \dfrac{\partial^2}{\partial x\partial z} & \dfrac{\partial^2}{\partial y\partial z} & \dfrac{\partial^2}{\partial z^2} \end{pmatrix}. \tag{34}$$

So in (32), (33) we obtain the analytical form for singular parts of the components of the tensor $\mathcal{E}$.

As it was mentioned above the Green tensor function is important in the theory of the integral equation method, where the solution of the Maxwell vector equation in an infinite space with bounded inhomogenuous domains can be reduced to the solution of the singular integral equations in the bounded domains. The kernel of the integral equations is a Green tensor.

The obtained hypersingular integral equation can be solved numerically. The local domain V is decomposed into subdomains V_i, where subdomains are assumed so small, that inside V_i the regular function can be taken for constant one:

$$\int\limits_{V_i}\mathcal{E}(\mathbf{R},\mathbf{R}')\mathbf{f}(\mathbf{R}')d\mathbf{R}' \simeq \int\limits_{V_i}\mathcal{E}(\mathbf{R},\mathbf{R}')d\mathbf{R}'\cdot\mathbf{f}(\mathbf{R}_0'), \quad \mathbf{R}_0'\in V_i. \tag{35}$$

If $\mathbf{R}\in V_i$, the integral in (35) will be singular with non-integrable singularity. Let us rewrite this integral as

$$\int\limits_{V_i}(\mathcal{E}-\mathcal{E}^0)d\mathbf{R}' + \int\limits_{V_i}\mathcal{E}^0 d\mathbf{R}'.$$

The integrand in the first integral is bounded or has a weak singularity and can be integrated numerically for example in spherical or cylindrical co-ordinates.

Let us write $\mathcal{E}^0$ (33) in the form

$$\mathcal{E}^0 = \frac{i\omega\mu_0}{4\pi}\frac{e^{ik_pR}}{R}\mathcal{I} + \frac{1}{4\pi\sigma_p}(\nabla\cdot\nabla^T)\frac{e^{ik_pR}-1}{R} + \frac{1}{4\pi\sigma_p}(\nabla\cdot\nabla^T)\frac{1}{R}. \tag{36}$$

The first term has the first order singularity and can be integrated easily. In the second term $(e^{ik_pR}-1)/R$ for small R is analytical function.

Let the subdomain V_i be symmetrical and let be

$$\mathcal{G}^0 = \int\limits_{V_i} \frac{1}{4\pi\sigma_p}(\nabla\cdot\nabla^T)\frac{1}{R}d\mathbf{R}'.$$

It is easy to see that the terms in $\mathcal{G}^0$ with non-diagonal differential operators from (34) are equal to zero, and we have

$$\mathcal{G}^0 = \frac{1}{4\pi\sigma_p}\begin{pmatrix} a_x & 0 & 0 \\ 0 & a_y & 0 \\ 0 & 0 & a_z \end{pmatrix}, \tag{37}$$

where

$$a_\alpha = \int\limits_{V_i} \frac{\partial^2}{\partial\alpha^2}\left(\frac{1}{R}\right)d\mathbf{R}', \quad \alpha = x, y, z. \tag{38}$$

Note the other peculiarity of the integrand

$$(\nabla_{\mathbf{R}}\cdot\nabla^T_{\mathbf{R}})\frac{1}{R} = (\nabla_{\mathbf{R}'}\cdot\nabla^T_{\mathbf{R}'})\frac{1}{R}.$$

For an analytical calculation of the integral (37) a usual method is using Cauchy's principal value for a singular integral, where for integrating a small domain is separated in the form of ball with centre in $\mathbf{R}$. However this choice is not convenient, because for a numerical solution the domain usually is decomposed into small parallelepipeds.

We give an analytical formula for a principle value of the hypersingular integral in the parallelepiped with sides h_x, h_y, h_z. We have

$$a_x = \int\limits_{-h_z/2}^{h_z/2} dz \int\limits_{-h_y/2}^{h_y/2} dy \int\limits_{-h_x/2}^{h_x/2} \frac{\partial^2}{\partial x^2}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)dx =$$

$$= -h_x \int\limits_{-h_z/2}^{h_z/2} dz \int\limits_{-h_y/2}^{h_y/2} \frac{dy}{(h_x^2/4 + y^2 + z^2)^{3/2}} = \tag{39}$$

$$= -8h_x h_y \int\limits_{-h_z/2}^{h_z/2} \frac{dz}{(h_x^2 + z^2)\sqrt{h_x^2 + h_y^2 + 4z^2}} = -8\mathrm{arctg}\frac{h_y h_z}{h_x\sqrt{h_x^2 + h_y^2 + h_z^2}}.$$

Analogously,

$$a_y = -8\mathrm{arctg}\frac{h_x h_z}{h_y\sqrt{h_x^2 + h_y^2 + h_z^2}}, \quad a_z = -8\mathrm{arctg}\frac{h_x h_y}{h_z\sqrt{h_x^2 + h_y^2 + h_z^2}}.$$

For the cube ($h_x = h_y = h_z = h$) we obtain

$$a_x = a_y = a_z = -8\mathrm{arctg}\frac{1}{\sqrt{3}} = -\frac{4\pi}{3}. \tag{40}$$

Note, that the formula (40) does not depend on h and so it is the same for arbitrary small h. Note, farther, that this last result is the same as it for ball, i.e. for Cauchy's principle value ([3]).

The above stated algorithm can be used efficiently for a fast calculation of the Green tensor in the subdomains which are more suitable for numerical calculations in parallelepipeds and cubes, therefore the integral equation method can be used more widely and successfully.

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R.H. Farzan
Széchenyi István Egyetem
Fizika Tanszék
Egyetem tér 1.
H-9007 Győr, Hungary

В.И. Дмитриев
МГУ, Факультет ВМК
Кафедра мат. физики
Ленинские горы
119992 Москва, Россия