BUILDING INFLUENCE NETWORKS IN THE FRAMEWORK OF POSSIBILITY THEORY

Pascale Fonck
Institut de Mathmématique,Université de Liège
15, avenue des Tilleuls - D1, B-4000 Liège, Belgium

Ewa Straszecka
Institute of Electronics,Silesian Technical University
16, Pstrowskiego St.44100 Gliwice, Poland

Abstract: The aim of this paper is to show the application of an influence network to a medical situation: the diagnosis of thyroïdism.
Here we will make the assumption that all uncertain informations are expressed using possibility and necessity meanses.
Thus we are using on extension of Pearl's work about influence networks.

Keywords: Uncertainty, influence network, possibility measure

1. THE BUILDING OF THE NETWORK

Let us denote by $A, B, C \cdots$ variables taking values denoted by $1, 2, 3, \cdots$.

The event A_i is the event "the variable A takes the value i".

The building of the network is based upon the relations of influence between the variables.

The variables are represented by the nodes in the network and the influences between the variables are represented by the links in the network.

The strength of these influences is expressed by the valuations assigned to the links.

We first choose an order $X^1, \cdots, X^n$ among the variables.

Then, for each variable X^i, we calculate the set $S^i \subseteq \{X^1, \cdots, X^{i-1}\}$ of variables which directly influence X^i.

Then we build links from the nodes belonging to S^i, to X^i, and we assign to these links numbers representing the strength of the dependences.

In this way, we obtain a network which contains all our knowledge about the dependences between the variables, and the strength of these dependences. We will also assume that we know the "a priori" degrees of possibility and necessity related to the nodes without parents in the network.

Then our problem is to compute the degrees of possibility and necessity related to the other nodes, that is to fuse all the data included in the network and to express the uncertainty of one single event A_i, for each variable A in the network.

2. FUSION OF THE DATA

Let us suppose that the network (Figure 1.) is singly connected, and contains the following part.

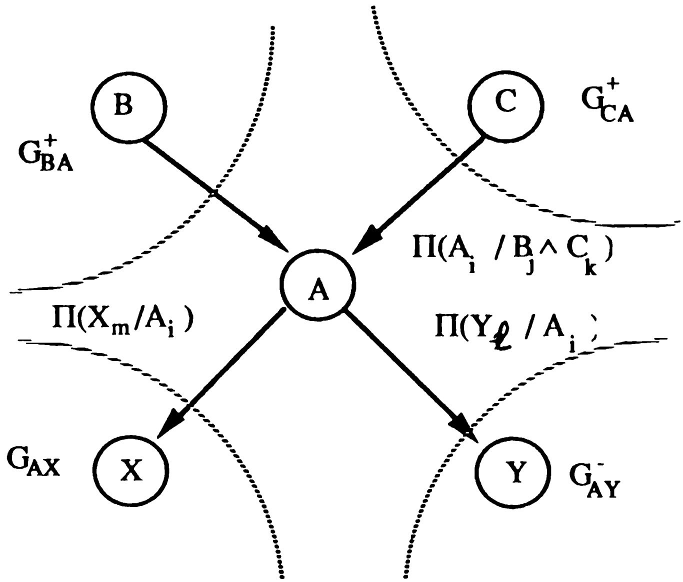

Figure 1.

This network is divided into four parts:

$$G^-_{BA}, G^+_{CA}, G^-_{AX} \text{ and } G^-_{AY}.$$

We will denote by $P^+_{BA}, D^+_{CA}, D^-_{AX}$ and D^-_{AY} the sets of data included in these subgraphs.

Then the degree of possibility $\pi(A_i/D^+_{BA} \wedge D^+_{CA} \wedge D^-_{AX} \wedge D^-_{AY})$ that the variable A takes the value i can be calculated in the following way:

1) to each link $U \to V$ we assign two vectors $\lambda_V(U)$ and $\mu_U(V)$.

   These vectors are defined by:

$$\lambda_V(U_i) = \pi(D^-_{UV}/V_i),$$

and

$$\mu_V(U_i) = \pi(U_i/D^+_{UV}),$$

for each value i of the variable U. The computation of these vectors is made as explained by steps 2), 3) and 4).

2) if V is a leaf node (a node without child), then

$$\lambda_V(U) = (1, \cdots, 1)$$

3) if U is a root node (a node without parent), then:

$$\mu_V(U) = (\pi(U_1), \cdots, \pi(U_n)),$$

   where $1, \cdots, n$ denote the possible values of the variable U.

4) from the values of these vectors, we compute the values of all vectors in the network in the following way:

$$\mu_X(A_i) = \text{Min}(\lambda_Y/A_i), \underset{j,k}{\text{Max}}\, \text{Min}(\pi(A_i/B_j \wedge C_k), \mu_A(B_j), \mu_A(C_k))) \quad (1)$$

$$\lambda_A(B_j) = \underset{k}{\text{Max}}\, \text{Min}(\mu_A/C_k), \underset{i}{\text{Max}}\, \text{Min}(\lambda_X(A_i), \lambda_Y(A_i), \pi(A_i/B_j \wedge C_k))). \quad (2)$$

5) when all these vectors are computed, the degree of possibility that the variable A takes the value i is computed by:

$$\pi(A_i) = \begin{cases} \text{Min}(\mu_X(A_i), \lambda_X(A_i)) & \text{if } i \text{ is such that} \\ & \exists j\text{: Min}(\mu_X(A_i), \lambda_X(A_i)) \\ & < \text{Min }(\mu_X(A_j), \lambda_X(A_j)) \\ 1 & \text{otherwise.} \end{cases}$$

3. APPLICATION

This method of data fusion can be applied to the case of the diagrams of thyroïdism.

We will suppose that we know the results of tests done by the patient.

So we know which symptoms a patient presents and which ones he does not present.

On the other hand, we know some factors expressing the importance of the presence or the absence of a symptom for the diagnosis of thyroïdism.

We will represent those data by the following network:

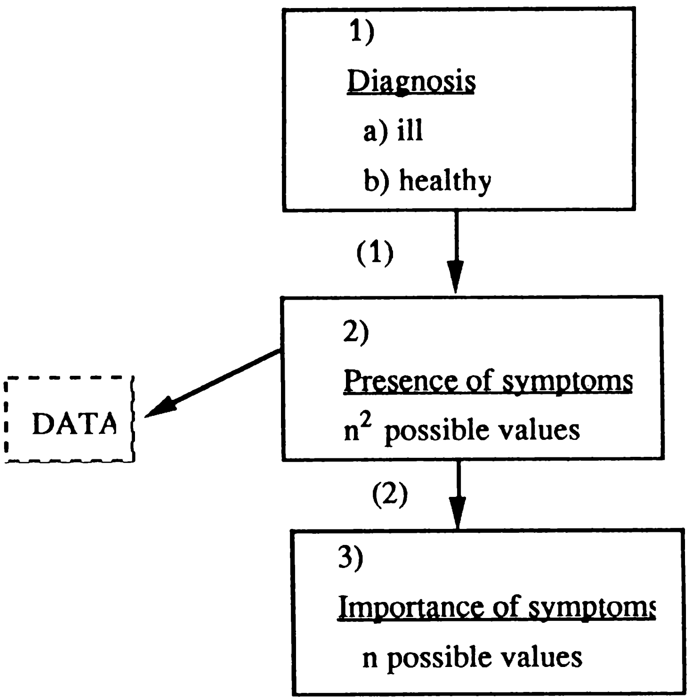

Figure 2.

We will assume that we know two a priori degrees of possibility

$$\pi(\text{ill}) \text{ and } \pi(\text{healthy}),$$

assigned to the first node.

Making the assumption that the symptoms are independent, we assign to the link (1) the numbers

$$\pi(S_i = a_1 \wedge \cdots \wedge S_n = a_n/ill) = \operatorname*{Min}_{1\leq i\leq n} \pi(S_i = a_i/ill)$$

where

$$a_i = 0 \text{ or } 1, \ \forall i \in \{1, \cdots, n\}$$

$$S_i = 1 \text{ means "the symptom } i \textit{ is absent"}$$

$$S_i = 1 \text{ means "the symptom } i \textit{ is present"}$$

To the link (2) we assign the numbers

$$\pi(S_j \text{ important } /S_j \text{ present})$$

and

$$\pi(S_j \text{ important } /S_j \text{ absent}),$$

for each symptom j.

The data are informations about the presence of symptoms for a given patient.

We will represent these informations by a new node, whose name is "DATA", which is a son of the node "PRESENCE OF SYMPTOMS".

This information is in fact the data of an n-tuple $(a_1, \cdots, a_n)$.

This information is represented by assigning the value of the vectors

$$\mu_{DATA} \text{ (Pres. of symptom) and } \lambda_{DATA} \text{ (Pres. of symptom)}$$

to:

$$\begin{cases} \mu_{DATA}(a_1, \cdots, a_n) = \lambda_{DATA}(a_1, \cdots, a_n) = 1 \\ \mu_{DATA}(b_1, \cdots, a_n) = \lambda_{DATA}(b_1, \cdots, b_n) = 0 \end{cases}$$

$$\text{if } (b_1, \cdots, b_n) \neq (a_1, \cdots, a_n).$$

The introduction of this new link and the new vector μ_{DATA} (presence of symptom) will induce changes for the vectors μ and λ carried by the links.

These changes are given by the relations (1) and (2).

In this case, the vectors which will successively be changed are:

$$\lambda_{PRES.\ OF\ SYMPT.}(DIAGNOSIS), \mu_{IMP.\ OF\ SUMPTOMS}(Pres.of sympt.).$$

After these computations, all λ and μ vectors will be calculated, and we will be able to deduce the value of the possibility degree:

$$\pi(ill) = \mathrm{Min}(\mu_{Pres.of\,sympt.}(ill), \lambda_{Pres.of\,sympt.}(ill))$$

and

$$\pi(healthy) = \mathrm{Min}(\mu_{Pr.of\,sympt.}(healthy), \lambda_{Pres.of\,sympt.}(healthy))$$

In this way, we can get informations about the certainty of the diagnosis.

The introduction of the node "PRESENCE OF SYMPTOMS" has been motivated by the Crook's test, after used in the diagnosis of thyroïdism.

The results given by this method still have to be tested and their fitting to already known diagnosis have to be observed.

REFERENCES

[1] Dubois D., Prade H. *Théorie des Possibilités. Application à la représentation des connaissances en informatique* 2e édition, (Masson, Paris, 1987)

[2] Pearl J. Fusion, Propagation and Sturcturing in Belief Networks, *Artificial Intelligence*, **29** (1986), 241-288.