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A FUZZY MOLP METHOD WITH GRAPHICAL DISPLAY OF FUZZINESS

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Abstract: The paper presents a visual interactive method, called 'FLIP', for solving MOLP problems with fuzzy coefficients in the objective functions and on the both sides of the constraints. The idea of 'FLIP' relies on an observation that for given values of decision variables the main question to be answered consists in the comparison of objectives and goals, and left- and right-hand-sides of the constraints which are fuzzy numbers. In result of application of a comparison principle, the fuzzy MOLP problem is transformed into a non-fuzzy multiobjective linear fractional programming (MOLFP) problem which is solved using an interactive sampling method. An evaluation of the quality of successive proposals (solutions) is based on the following characteristics: (i) scores of fuzzy objectives in relation to the goals, (ii) dispersion of values of the fuzzy objectives due to uncertainty, (iii) safety of solutions or risk of violation of the constraints. The graphics displayed by 'FLIP' provide the most compreshensive synthesis of these characteristics.

Keywords: Fuzzy linear programming, multiple objectives, visual interaction, graphical display of fuzziness

1. INTRODUCTION

In recent years, we have been able to observe a growing interest in multiobjective linear programming (MOLP) under uncertainty and imprecision. This follows from the fact that deterministic multiobjective optimization models are often insufficient in practical situations, especially in long-term planning problems, development strategies and agricultural decision problems. The coefficients that appear in MOLP problems may not be well-defined, either because their value depends upon other parameters (not accounted for in the model) or because they cannot be precisely assessed, and only qualitative estimates of these coefficients are available. Moreover, the constraints expressed by equalities or inequalities between linear expressions are often softer in reality than what their mathematical expression might let us believe.

This situation has motivated a search for more flexible formulations of optimization models that, although remaining rigorous, may help bridging the gap between the mathematical models and the real decision-making situations through handling of uncertainty and imprecision. Two distinct lines of research try to address these issues: stochastic linear programming and fuzzy linear programming that have developed independently. The first comparative study of stochastic and fuzzy approaches to MOLP has been performed by Słowiński and Teghem [9]. Recently, a more comprehensive analysis of both approaches has been described in a multi-author book edited by Słowiński and Teghem [10]. Being written by leading specialists of each approach, this is the most complete source book on multiobjective mathematical programming under uncertainty and imprecision.

In this paper, we are dealing with a method, called 'FLIP', representing the fuzzy approach to modelling of uncertainty and imprecision. Its theoretical foundations have been described first in Słowiński [6] and then developed in Słowiński [8]. It has been applied for solving many real-life decision problems, in particular, water supply planning problem (Słowiński [6]; Słowiński at al. [11]; Słowiński [7]), diet optimization for farm animals (Czyżak [1]; Czyżak, Słowiński [3]; Czyżak, Słowiński [4]) and farm structure optimization (Czyżak [2]). In all these applications, the experts taking part in model building have accepted to express uncertain or imprecise parameters in terms of tolerance intervals with a most possible value (or subinterval) and decreasing possibility for other values within the interval. This corresponds exactly to the definition of fuzzy numbers, i.e. normal convex continuous fuzzy subsets of the real line. So, the modelling of uncertainty and imprecision using fuzzy numbers was quite natural there.

The aim of the present paper is to characterize the most recent microcomputer implementation of 'FLIP' with an emphasis on the visual interactive aspect of the method. This presentation will be developed in the next paper by Czyżak and Słowiński ([5]).

2. FOUNDATIONS OF THE 'FLIP' METHOD

'FLIP' solves the following MOLP problem with fuzzy coefficients:

$$\widetilde{Maximize} \begin{bmatrix} \tilde{z}_1 & = & \underline{\tilde{c}}_1 \underline{x} \\ & \vdots & \\ \tilde{z}_k & = & \underline{\tilde{c}}_k \underline{x} \end{bmatrix} \tag{1}$$

$$\text{s.t.} \qquad \underline{\tilde{a}}_i \underline{x} \lessapprox \tilde{b}_i \qquad i = 1, \ldots, m_1 \tag{2}$$

$$\underline{x} \geq \underline{0} \tag{3}$$

where $\underline{x}$ is a column vector of n decision variables, $\tilde{\underline{c}}_1, \ldots, \tilde{\underline{c}}_k$ are row vectors of fuzzy cost coefficients corresponding to the objective functions $\tilde{\underline{z}}_1, \ldots, \tilde{\underline{z}}_k$, $\tilde{\underline{a}}_i$ is the i-th row of the matrix of fuzzy coefficients, and $\tilde{b}_i$ is its corresponding fuzzy right-hand-side. Notice that a "greater then or equal to" constraint can be transformed to (2) by multiplying the constraint by -1, and equality constraint can be presented as a pair of weak inequality constraints with opposite relation signs. It is assumed, moreover, that for the particular objectives, the decision maker (DM) is in a position to define fuzzy aspiration levels, thought of as goals, denoted by $\tilde{g}_1, \ldots, \tilde{g}_k$.

All fuzzy coefficients are given as L-R type fuzzy numbers, i.e. number $\tilde{a}$ is a triple of parameters (a,α,β) of its membership function

$$\mu_a(x) = \begin{cases} L((a-x)/\alpha) & \text{if } x \leq a \\ R((x-a)/\beta) & \text{if } x \geq a \end{cases}$$

where a is an interval of the "most possible" values, α and β are nonnegative left and right "spreads" of $\tilde{a}$, respectively, and L, R are symmetric bell-shaped reference functions that are decreasing in $(-\infty, \infty)$ and $L(0) = R(0) = 1$, $L(1) = R(1) = 0$; $\tilde{a}$ is said to be an L-R fuzzy number. When the spreads are zero, then $\tilde{a}$ is a nonfuzzy (crisp) number equal to a.

The idea of FLIP relies on an observation that for a given $\underline{x}$, the main question to be answered consists in the comparison of the left- and right-hand-sides in objectives and constraints which are fuzzy numbers. Assuming an L-R representation of fuzzy coefficients, a comparison principle has been proposed ([6],[7]) which allows a transformation of the fuzzy MOLP problem into a multiobjective linear fractional programming problem. The best compromise solution of the latter problem ensures the "best consistency" between the goals and the objective functions, and satisfies the constraints with a given "safety".

Let us recall informally the comparison principle using an example of constraint i for a given $\underline{x}$. Fuzzy coefficients of the constraints are given as L-R fuzzy numbers:

$$\tilde{a}_i = (\underline{a}_i, \underline{\alpha}_i \underline{\beta}_i)_{LR}\ , \qquad \tilde{b}_i = (b_i, \gamma_i, \delta_i)_{LR}, \qquad i = 1, \ldots, m.$$

For the sake of clarity, we assume that the reference functions of all fuzzy coefficients are of two kinds only: L and R. This is not, however, a general assumption of the comparison principle (cf. [8]).

It should be specified that all arithmetic operations on fuzzy numbers taking place in (1)-(3) are extended operations in the sense of Zadeh's principle. For any $\underline{x} \geq \underline{0}$, the left-hand-side of the i-th constraint can be summarized to the fuzzy number:

$$\tilde{\underline{a}}_i \underline{x} = (\underline{a}_i \underline{x},\ \underline{\alpha}_i \underline{x},\ \underline{\beta}_i \underline{x})_{LR}\ , \qquad i = 1, \ldots, m.$$

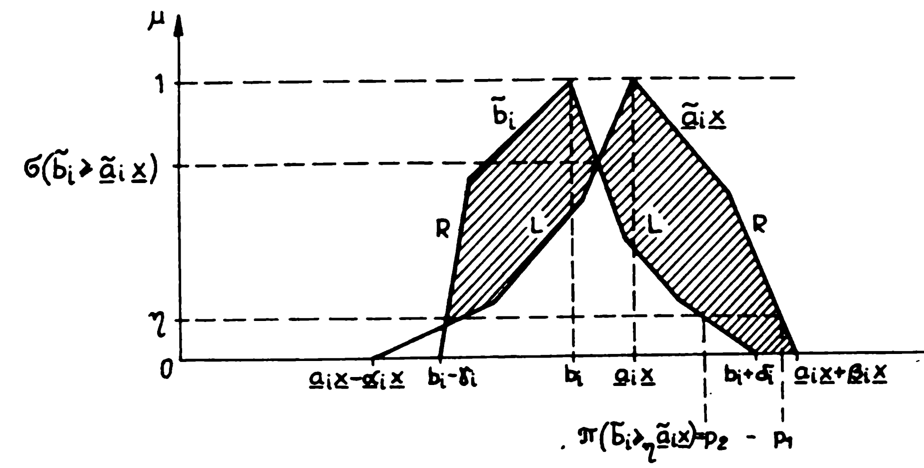

Fig.1. Fuzzy constraint $\tilde{\underline{a}}_i\underline{x} \leq \tilde{b}_i$ for a given $\underline{x}$.

Fig.1 shows relative positions of fuzzy numbers $\tilde{\underline{a}}_i\underline{x}$ and $\tilde{b}_i$.

In order to evaluate the degree of possibility for $\tilde{b}_i$ to be greater than or equal to $\tilde{\underline{a}}_i\underline{x}$, 'FLIP' uses two indices, one called optimistic (σ) and another pessimistic (π). The optimistic index $\sigma(\tilde{b}_i \geq \tilde{\underline{a}}_i\underline{x})$ is defined as an ordinate of the intersection point of the right slope of $\tilde{b}_i$ with the left slope of $\tilde{\underline{a}}_i\underline{x}$. Index σ is seen as optimistic because even if it takes value close to 1, the possibility of violation of the constraint $\tilde{\underline{a}}_i\underline{x} \leq \tilde{b}_i$ may be quite big. As a measure of this possibility one can consider the hatched area marked in Fig.1. Thus, in order to make the comparison more credible, one should use σ conjointly with the pessimistic index π.

Pessimistic index π follows from the comparison of the right slopes of $\tilde{\underline{a}}_i\underline{x}$ and $\tilde{b}_i$ at some level $0 \leq \eta \leq 1$. Specifically, $\pi(\tilde{b}_i \geq_\eta \tilde{\underline{a}}_i\underline{x}) = p_2 - p_1$.

Now, one can admit that $\tilde{b}_i \geq \tilde{\underline{a}}_i\underline{x}$ at credibility levels τ and η if and only if

$$\sigma(\tilde{b}_i \geq \tilde{\underline{a}}_i\underline{x}) \geq \tau \qquad \text{and} \qquad \pi(\tilde{b}_i \geq_\eta \tilde{\underline{a}}_i\underline{x}) \geq \theta \tag{4}$$

where $\tau,\eta \in [0,1]$ and $\theta \in (-\infty,\infty)$. $\theta \geq 0$ means that for any pair (v, y) such that $v \geq \underline{a}_i\underline{x}$, $y \geq b_i$ and $0 \leq \mu_{\underline{a}_i\underline{x}}(v) = \mu_{b_i}(y) \leq \eta$, inequality $y \geq v$ is true. A negative value of θ makes possible that inequality $y \geq v$ is not true.

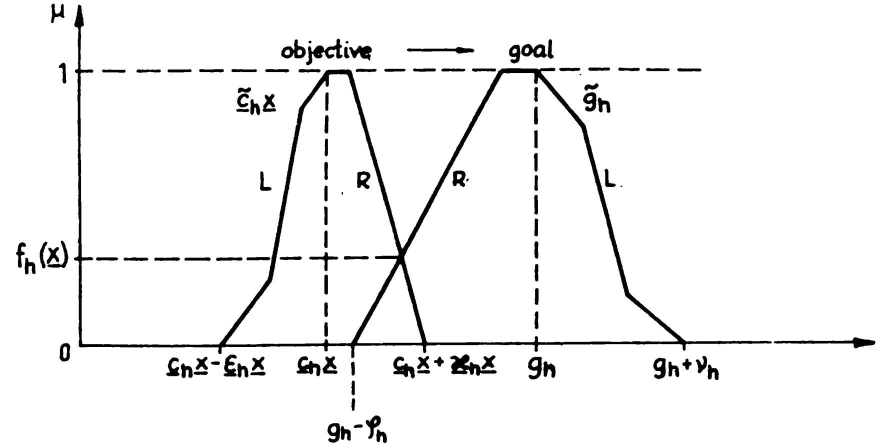

Fig.2. Fuzzy objective $\tilde{\underline{c}}_h\underline{x}$ and fuzzy goal $\tilde{g}_h$ for a given $\underline{x}$.

The constants τ,η and θ are called "safety parameters" because they are responsible for the safety of the assertion that $\tilde{b}_i$ is greater than $\tilde{\underline{a}}_i\underline{x}$. Let us remark that using τ,η and θ one can control the surface of the hatched area marked in Fig.1 which corresponds to the risk of violation of the constraint.

Thus, application of the above comparison principle transforms every fuzzy constraint (2) into two linear constraints corresponding to conditions (4) on σ and π, respectively (cf. [8]):

$$\underline{a}_i\underline{x} - b_i \le L^{-1}(\tau_i)(\underline{\alpha}_i\underline{x} + \delta_i) \qquad i = 1,\dots,m \tag{5}$$

$$b_i + \delta_i L^{-1}(\eta_i) - \underline{a}_i\underline{x} - \underline{\beta}_i\underline{x}R^{-1}(\eta_i) \ge \theta_i \qquad i = 1,\dots,m \tag{6}$$

In the case of fuzzy objective functions, σ can be used to evaluate the degree of consistency between fuzzy objectives and fuzzy goals. Let the fuzzy cost coefficients and fuzzy goals be:

$$\tilde{\underline{c}}_h = (\underline{c}_h, \underline{\epsilon}_h, \underline{\kappa}_h)_{LR}\ , \qquad \tilde{g}_h = (g_h, \phi_h, \nu_h)_{RL}\ , \qquad h = 1,\dots,k.$$

Here again, the equality of the reference functions is not a necessary assumption. For any $\underline{x}$, the components of the h-th objective function can be summarized with the extension principle to the fuzzy number:

$$\tilde{\underline{c}}_h\underline{x} = (\underline{c}_h\underline{x},\ \underline{\epsilon}_h\underline{x},\ \underline{\kappa}_h\underline{x})_{LR}, \qquad h = 1,\dots,k.$$

Fig.2 shows relative positions of fuzzy numbers $\tilde{\underline{c}}_h\underline{x}$ and $\tilde{g}_h$.

In order to maximize the consistency between $\tilde{c}_h \underline{x}$ and $\tilde{g}_h$ one has to maximize the ordinate $f_h(\underline{x})$ of the intersection point of the right slope of $\tilde{c}_h \underline{x}$ with the left slope of $\tilde{g}_h$. Mathematically,

$$f_h(\underline{x}) = R[(\underline{c}_h \underline{x} - g_h)/(\underline{\kappa}_h \underline{x} + \phi_h)], \qquad h = 1, \ldots, k \tag{7}$$

If reference function R of fuzzy cost coefficients and goals is linear (or piecewise linear), then (7) takes the fractional form:

$$f_h(\underline{x}) = 1 - (\underline{c}_h \underline{x} - g_h)/(\underline{\kappa}_h \underline{x} + \phi_h), \qquad h = 1, \ldots, k.$$

In consequence, for given goals and safety parameters, the fuzzy MOLP problem (1)-(3) is transformed into the following deterministic multiobjective programming problem:

$$Maximize \begin{bmatrix} \tilde{f}_1(\underline{x}) \\ \vdots \\ \tilde{f}_k(\underline{x}) \end{bmatrix}$$

s.t. (3), (5) and (6)

If $f_h(\underline{x})$ are defined by (8), this problem is a multiobjective linear fractional programming (MOLFP) one.

The associate deterministic MOLFP problem is solved using an interactive sampling procedure (cf. [8]). In each calculation step, a sample of efficient points of the MOLFP problem is generated and then shown to the decision maker (DM) who is asked to select the one that best fits his preferences. If the selected point is not the final compromise, it becomes a central point of an efficient region which is sampled in the next calculation step. In this way, the sampled part of the efficient border is successively reduced (focusing phenomenon). The interactive process ceases when the most satisfactory efficient point is reached.

An important advantage of the above algorithm is that the only optimization procedure to be used is a linear programming one. Moreover, it has a simple scheme and allows retractions to the points which have been found uninteresting in previous iterations.

3. INTERACTIVE STEPS OF 'FLIP'

'FLIP' can be summarized in the following steps:

Step 1. Formulation of problem (1)-(3) and definition of fuzzy coefficients.

Step 2. Definition of fuzzy aspiration levels $\tilde{g}_h$ $(h = 1, \ldots, k)$ on objectives.

Step 3. Definition of safety parameters τ_i , η_i and θ_i $(i = 1, \ldots, m)$ for fuzzy constraints.

Step 4. Formulation of the associate multiobjective deterministic problem (9), (3), (5), (6).

Step 5. Application of an interactive method for solving the associate deterministic problem formulated in step 4.

Step 6. If a best compromise solution has been found then stop, otherwise return to step 3 for revision of safety parameters. Retraction to steps 1 and 2 for redefinition of fuzzy coefficients and/or aspiration levels is also possible.

The DM intervenes in two steps of FLIP. First, when fixing the safety parameters (step 3), and then in the course of the guided generation and evaluation of the efficient points of the associate deterministic problem (step 5). Thus, the interaction with the DM takes place at two levels. As to the first one (step 3), it is worth noting that there are two practical ways of controlling the safety of solutions using parameters τ_i , η_i and θ_i :

(a) fix $\eta_i = 0$, $i = 1, \ldots, m$, and control the optimistic safety with τ_i ,and the pessimistic safety with θ_i, $i = 1, \ldots, m$, or

(b) fix $\theta_i = 0$, $i = 1, \ldots, m$, and control the optimistic safety with τ_i , and the pessimistic safety with η_i, $i = 1, \ldots, m$.

The safety parameters are defined taking into account their interval of variation and the knowledge acquired in preceding iterations about the dependency between safety and the quality of the compromise among criteria. If way (a) is chosen, 'FLIP' provides the DM with an information about an approximate interval of variation of θ_i at level $\eta_i = 0 (i = 1, \ldots, m)$.

In the microcomputer implementation of 'FLIP', the efficient points proposed to the DM are presented both numerically, in terms of $\underline{x}$ and middle values of $\tilde{z}_h(\underline{x})$, $h = 1, \ldots, k$, and graphically, in terms of mutual positions of fuzzy numbers corresponding to objectives and aspiration levels on the one hand, and to left- and right-hand-sides of constraints, on the other hand. In this way, the DM gets quite a complete idea about the quality of each proposed solution. The quality is evaluated taking into account the following characteristics:

- scores of fuzzy objectives in relation to the goals,
- dispersion of values of the fuzzy objectives due to uncertainty,
- safety of the solution or, using a complementary term, the risk of violation of the constraints (cf. the hatched areas in Fig.1).

So, the definition of the best compromise involves an analysis of the compromise among criteria and an evaluation of the safety of the corresponding solution.

The analysis of the above characteristics needs indeed a graphical display of objectives and constraints in a form similar to Figs. 1 and 2. We claim that the comparison of fuzzy left- and right-hand-sides of the constraints, as well as evaluation of dispersion of the values of objectives, is practically infeasible on the basis of one or two numerals. The graphical representation of proposed solutions is not only a "user's friendly" interface but the best way of a complete characterization of these solutions.

We put emphasis on this aspect of visual interaction because it is underestimated in all procedures proposed for solving multiobjective programming problems under uncertainty and imprecision.

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