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MULTIPOINT BOUNDARY VALUE PROBLEMS FOR CALCULATING THE SCREENING EFFECT OF A METAL CABLE SHEATH WITH NON-LINEARITY DUE TO STEEL AMOURING

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1. Introduction

The increasing interference between lines running in close vicinity is one of the consequences of the continuous growth of electric power network, electrified railway systems and telecommunication lines. This interference may result in danger to persons working on and using the telecommunication facilities, fault in the equipment and degradation in the transmission quality. An accurate analysis of the interference is difficult mainly due to the complexity and great number of influencing factors. Lately, great efforts have been made to develop sophisticated modelling of the problem reflecting the real physical phenomena. When an adequate algorithm is applied the problem can be solved utilizing the possibilities supplied by the computer techniques [1].

The inductive coupling is the most common form of interference. Its result can be limited to a reasonable level with appropriate screening given by the sheath of telecommunication cable. The screening effect can significantly be improved by the use of steel-tape armour around the metallic cable sheath. This ferromagnetic armour is excited by the induced sheath current resulting an additional impedance in the sheath-to-earth circuit causing its non-linear variation. In addition, the screening action is influenced by the earthing of the sheath which is realized by the distributed conductance to earth and discrete earthing applied at least at the line ends.

Traditionally, the screenig action is taken into consideration by the nominal screening factor. This is only applicable when the sheath is perfectly earthed, which is generally an unrealistic assumption is practice.

In this paper the screening action of a cable sheath will be studied, for any conditions in practice, solving the sheath-to-earth circuit. An equivalent transmission line model is given which is described by a system of differential equations with appropriate boundary conditons. The mathematical problem is solved by a numerical algorithm using a multiple shooting technique. This technique enables to consider the value of impedence due to steel armour in accordance with excitation of the actual magnitude of the sheath current. The calculations will primarily result in the distribution of the sheath current and the sheath-to-earth voltage. When the current is multiplied by the sheath resistance the longitudinal field strength (e.m.f.) affecting the telecommunication circuit is obtained. The results of computer simulations related to a given cable sheath, and various case and parametric studies will also be presented.

2. Principles of calculating the screening effect

When the results of inductive coupling is to be determined as a starting point, the longitudinal electromotive force (e.m.f.) E, per unit length, induced along the telecommunication line will be calculated. It is given by the product of the mutual impedance, per unit length, between the inducing and induced circuits and the inducing current. The value of E may vary with respect to the length coordinate x measured along the induced line due to the variation in either the mutual impedance (case of non-parallel lines) or the inducing current.

The telecommunication circuit in a metal-sheathed cable is actually affected by the e.m.f. E_s occuring inside the sheath (Fig.1a). Its value can be expressed as

(2.1)

$$E_s(x) = R_{dc} \cdot I(x)$$

where R_{dc} is the d.c. resistance, per unit length, of the sheath and I(x) is the sheath current at location x. For a cable sheath having continuous leakage to the earth the calculation of E_s can be performed by either of the two ways hereunder.

a. Calculation by the sceening factor

Assuming a perfectly earthed sheath, the following simple relationship exists between E(x) and I(x):

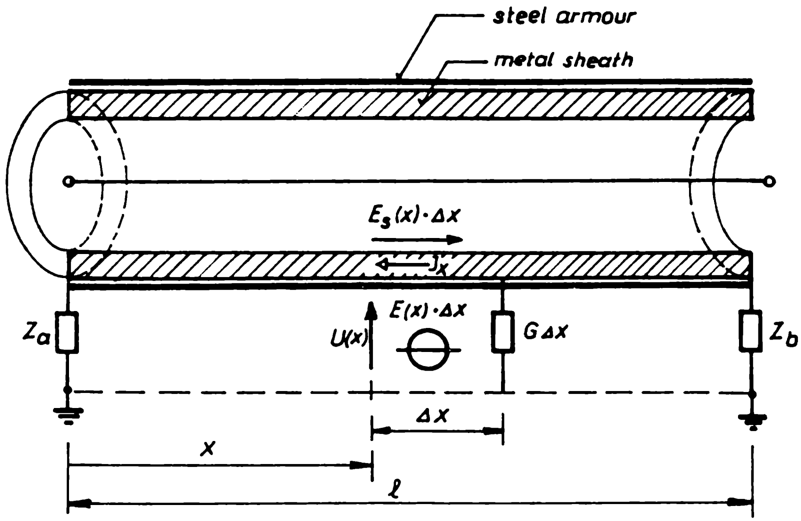
(2.2)

$$E(x) = Z \cdot I(x)$$

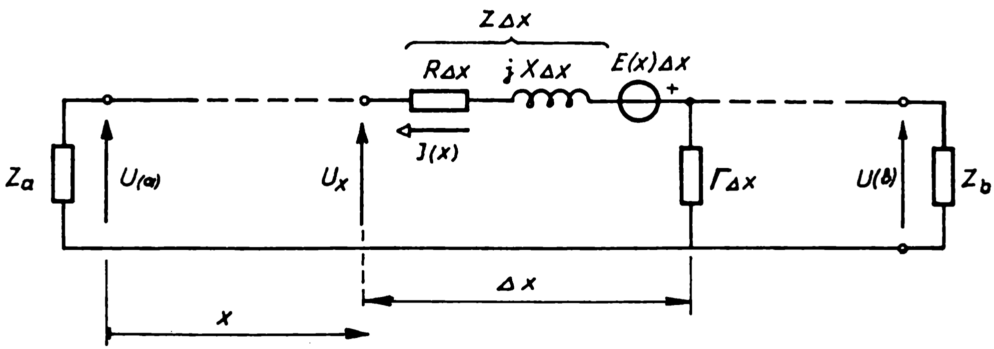
where Z is the series impedance, per unit length, of the sheath-to-earth circuit. Substituting the sheath current from (2.2) into (2.1) yields

(2.3)

$$E_s(x) = \frac{R_{dc}}{Z} E(x).$$



a. system considered



b. equivalent distributed line circuit

Fig.1. Representation of the sheath-to-earth circuit of an induced cable

Defining the sceening factor k of the sheath as

(2.4a)

$$k = \frac{E_s(x)}{E(x)} = \frac{R_{dc} \cdot I(x)}{E(x)}$$

we get from (2.3) for the nominal screening factor k_N

(2.4b)

$$k_N = \frac{R_{dc}}{Z}.$$

So the screening factor k can be calculated from the parameters of the sheath-to-earth circuit. On the other hand the e.m.f. $E_s(x)$ with the consideration of the screening effect is given by

(2.5)

$$E_s(x) = k_N \cdot E(x).$$

For a steel-armoured cable, Z depends on the magnitude of the sheath current (see Fig.2), i.e. it reads $Z(\mid I \mid)$. Consequently, considering (2.2) and (2.4b) the screening factor can be expressed as a function of $\mid E \mid$, i.e. it reads $k(\mid E \mid)$.

In order to determine $E_s(x)$, using the screening factor, we take the following steps: (1) calculate E(x); (2) take the value of $k(\mid E \mid)$ to the relevant $\mid E \mid$; (3) calculate $E_s(x)$ by (2.5).

<u>Calculation by the solution of the sheath-to-earth circuit</u>

When the sheath-to-earth circuit is not perfectly earthed, (2.2) does not applicable any more. Thus, to obtain the sheath current I(x) the sheath-to-earth circuit equations should be solved. This circuit can be represented by a two-wire transmission line excited by a distributed e.m.f. E (Fig.1b). For the sheath-to-earth voltage U(x) and sheath current I(x) the following system of differential equations can be written:

$$\frac{dU(X)}{dx} = Z \cdot I(x) + E(x),$$

(2.6)
$$\frac{dI(x)}{dx} = \Gamma \cdot U(x),$$

where Γ is the distributed shunt admittance of the sheath-to-earth circuit, which represents the conductance G, per unit length, of the leakage between the sheath and the earth. Its value is assumed to be uniform along the line.

It is worth mentioning that U(x) and I(x) are complex functions and they are replaced by their real and imaginary parts for the numerical algorithm:

(2.7)
$$U(x) = U_r(x) + jU_i(x),$$
$$I(x) = I_r(x) + jI_i(x),$$

where j is the imaginary unit, so the system (2.6) results in a system of four equations with real functions.

Z, the distributed series impedance of the circuit is function of the modulus of the current I(x), i.e. of

(2.8)
$$\mid I \mid = \sqrt{I_r^2 + I_i^2},$$

and can be written as

(2.9)
$$Z(\mid I \mid) = R(\mid I \mid) + jX(\mid I \mid),$$

where the resistance R and reactance X are taken in Ω/km, namely

(2.10)
$$R(\mid I \mid) = 0.05 + R_k(\mid I \mid),$$
$$X(\mid I \mid) = 0.579 + X_k(\mid I \mid).$$

The values 0.05 and 0.579 are the resistance and reactance of the earth return path at 50 Hz, respectively, while R_k and X_k are the resistance and reactance of the outer surface impedance of the sheath with external (earth) current return.

They include the additional impedance due to steel armour which depends on the sheath current. The magnitude of R_k and X_k are known from measurements at discrete current values, being used as prescribed values of cubic spline functions which are to be determined in the numerical algorithm (see 3).

The current dependence of R and X results in nonlinearity of the system (2.6).

The induced e.m.f. E(x) is assumed to be known in complex form as

(2.11)

$$E(x) = E_r(x) + jE_i(x).$$

In practical cases, the exposures are represented by equivalent paralelism, thus E(x) is given in form of stepwise constant function with respect to x.

The boundary conditions are given by the voltage current relations, i.e. Kirchoff's equations on the left-hand termination (x=a) and on the right-hand termination (x=b):

(2.12)

$$U(a) - Z_a \cdot I(a) = 0,$$
$$U(b) + Z_b \cdot I(b) = 0,$$

where Z_a and Z_b are the complex earthing impedances at the relevant ends of line (Fig.1b). The boundary conditions are also replaced by their real and imaginary parts for the numerical algorithm.

3. Mathematical modelling

The mathematical model of the problem presented in 2 can be written by the following system of N differential equations,

(3.1)

$$Y'(x) = F(x, Y(x), G(H(Y(x))), S(x)), \quad x \epsilon [a,b] \subset R$$

where

$$Y(x) = \begin{pmatrix} Y_1(x) \\ \vdots \\ Y_N(x) \end{pmatrix} \in R^N$$

is the function to be determined on the interval [a,b],

$$H(Y) = H(Y_1, \ldots, Y_N) \in R$$

is an explicitly given function of N variables,

$$G(H) = \begin{pmatrix} G_1(H) \\ \vdots \\ G_l(H) \end{pmatrix} \in R^l$$

is a function given at certain discrete values of H (measurements),

$$S(x) = \begin{pmatrix} S_1(x) \\ \vdots \\ S_v(x) \end{pmatrix} \in R^v$$

is a piecewise constant function given on the interval [a,b],

$$F(x, Y, G, S) = \begin{pmatrix} F_1(x, Y, G, S) \\ \vdots \\ F_N(x, Y, G, S) \end{pmatrix} \in R^N$$

is the right-hand side of the differential equation (3.1) supposed to be sufficiently smooth function of $(N + l + v + 1)$ variables,

satisfying the following linear and separated boundary conditions (3.2)

$$AY(a) = c, \quad BY(b) = d,$$

with given matrices $A \in R^{p \times N}, B \in R^{q \times N}$ and given vectors $c \in R^p, d \in R^q$, for which the numbers p and q are assumed to satisfy the relation p+q=N.

Here the function Y(x) corresponds to the real and imaginary parts of the voltage U(x) and the current I(x), such that

$$Y_1(x) := U_r(x), \quad Y_3(x) := I_r(x),$$

$$Y_2(x) := U_i(x), \quad Y_4(x) := I_i(x),$$

therefore N=4,

H(Y) is given by (2.8) as $| I |$, *i.e.* $H(Y) := \sqrt{Y_3{}^2 + Y_4{}^2}$, $G(H)$ corresponds to the real and imaginary parts of the complex function Z figuring in (2.9) and (2.10) given by measurements at discrete values of the current I, that is

$$G_1(H) := 0.05 + R_k(H), \quad G_2(H) := 0.579 + X_k(H),$$

consequently l=2,

S(x) includes the e.m.f. components $E_r(x)$ *and* $E_i(x)$ appearing in (2.11), so v=2, i.e.

$$S_1(x) := E_r(x), \quad S_2(x) := E_i(x).$$

The system of differential equations (3.1) corresponds to the system given in (2.6) and written into their corresponding real form, that is

$$Y_1{}'(x) = G_1(H(Y(x))) \cdot Y_3(x) - G_2(H(Y(x))) \cdot Y_4(x) + S_1(x),$$

$$Y_2{}'(x) = G_2(H(Y(x))) \cdot Y_3(x) + G_1(H(Y(x))) \cdot Y_4(x) + S_2(x),$$

$$Y_3{}'(x) = \lambda_1 \cdot Y_1(x) - \lambda_2 \cdot Y_2(x),$$

$$Y_4{}'(x) = \lambda_2 \cdot Y_1(x) + \lambda_1 \cdot Y_2(x),$$

where λ_1 *and* λ_2 are given constants.

Concerning the boundary conditions (3.2) they are given by their complex forms in (2.12), so p=2 and q=2 for their corresponding real forms.

Now we transform the problem as follows. Through the introduction of v additional functions and differential equations

$$\begin{aligned} Y_{N+1}(x) &:= S_1(x), & Y'_{N+1}(x) &= 0, \\ &\vdots & &\vdots \\ Y_{N+v}(x) &:= S_v(x), & Y'_{N+v}(x) &= 0, \end{aligned}$$

the problem (3.1) - (3.2) is seen to be equivalent to the following multipoint boundary value problem of dimension n=N+v:

(3.3)

$$y'(x) = f(x, y(x)), \quad x \in [a, b] \subset R, \qquad y(x) \in R^n,$$

with linear and separated boundary conditions of the form

(3.4)

$$A^* y(a) = c^*, \quad B^* y(b) = d^*,$$

(3.5)

$$A_i^- y(x_i^-) + A_i^+ y(x_i^+) = s_i, \quad i = 2, 3, \ldots, M-1,$$

where

$$y(x) = \begin{pmatrix} y_1(x) \\ \vdots \\ y_n(x) \end{pmatrix} := \begin{pmatrix} Y_1(x) \\ \vdots \\ Y_N(x) \\ Y_{N+1}(x) \\ \vdots \\ Y_{N+v}(x) \end{pmatrix},$$

$$f(x, y) = \begin{pmatrix} f_1(x, y) \\ \vdots \\ f_n(x, y) \end{pmatrix} :=$$

$$:= \begin{pmatrix} F_1(x, y_1, \ldots, y_N, G(H(y_1, \ldots, y_N)), y_{N+1}, \ldots, y_{N+v}) \\ \vdots \\ F_N(x, y_1, \ldots, y_N, G(H(y_1, \ldots, y_N)), y_{N+1}, \ldots, y_{N+v}) \\ 0 \\ \vdots \end{pmatrix}$$

$$A^* := \begin{pmatrix} A & O \\ 0 & I_v \end{pmatrix} \in R^{(p+v)\times n}, \quad c^* := \begin{pmatrix} c \\ S_1(a) \\ \vdots \\ S_v(a) \end{pmatrix} \in R^{p+v},$$

$$B^* := (B \quad 0) \in R^{q\times n}, \qquad d^* := d \in R^q,$$
$$A_i^- := -I_n \in R^{n\times n}, \qquad A_i^+ := I_n \in R^{n\times n},$$

$$s_i := \begin{pmatrix} 0 \\ \vdots \\ 0 \\ S_1(x_i^+) - S_1(x_i^-) \\ \vdots \\ S_v(x_i^+) - S_v(x_i^-) \end{pmatrix} \in R^n, \qquad i = 2, 3, \ldots, M-1,$$

x_i are the points of the interval [a,b] where the function S(x) has discontinuity (jump increase or decrease),
$y(x_i^+)$, $y(x_i^-)$, $S_j(x_i^+)$, $S_j(x_i^-)$ are the usual right and left limits at $x = x_i$,
I_n is the $n \times n$ identity matrix.

We assume that f is twice continuously differentiable with respect to y in the regions $\Omega_i := [x_i, x_{i+1}] \times R^n$, and continuous in x on the subintervals $[x_i, x_{i+1}]$ between the boundary points. We suppose that the problem (3.3) - (3.4) - (3.5) has a solution, which we denote by $y^*(x)$.

4. Numerical algorithm

In order to solve the problem (3.3) - (3.4) - (3.5) we use a multiple shooting technique [1.]. To compute the right-hand side $f(x,y)$ of the differential equation (3.3) and its partial derivatives at any interior points of [a,b] we use third order spline functions [2] (for computing the function G, which is given at discrete points only), which are determined before beginning the interative cycles of the shooting method.

To prepare the interations let us first choose intermediate points x_j in [a,b] so that they will be, together with the given points x_i of discontinuity, suitable for the convergence of the multiple shooting technique.

Let us order and re-label the points a, b, x_i and the chosen points x_j as $a = t_1 < t_2 < \ldots < t_m = b$, and consider t as the new independent variable. We assume initial approximations ${}^1y(t_j^+)$ to be given for all these points except the last one, i.e. for $j = 1, 2, \ldots, m-1$.

At the k-th step of the procedure we solve the following initial value problems

$$(4.1) \qquad y'(t) = f(t, y(t)), \qquad t \in [t_j, t_{j+1}],$$

$$(4.2) \qquad y(t_j) = {}^k y(t_j^+), \qquad j = 1, 2, ..., m-1,$$

and denote the solutions by ${}^k y_j(t), \ \ j = 1, 2, ..., m-1$.
Let us introduce the error functions
(4.3)

$$ {}^k\eta_j(t) := y^*(t) - {}^k y_j(t), \qquad t \in [t_j, t_{j+1}], \ j = 1, 2, ..., m-1,$$

for which we have
(4.4)

$$ {}^k\eta_j'(t) = f(t, y^*(t)) - f(t, {}^k y_j(t)), \qquad t \in [t_j, t_{j+1}], \ j = 1, 2, ..., m-1.$$

Then consider the following initial value problems

(4.5) $\quad \eta^{\cdot}(t) = D_y f(t, {}^k y_j(t))\eta(t), \qquad t \in [t_j, t_{j+1}],$

(4.6) $\quad \eta(t_j) = y^*(t_j) - {}^k y_j(t_j), \qquad j = 1, 2, ..., m-1,$

the solutions of which approach the functions (4.3) according to truncated Taylor's series expansions about $(t, {}^k y_j(t))$ in (4.4).
Now let us solve the initial value problems of the following matrix differential equations

(4.7) $\quad \Psi^{\cdot}(t) = D_y f(t, {}^k y_j(t))\Psi(t), \qquad t \in [t_j, t_{j+1}],$

(4.8) $\quad \Psi(t_j) = I_n, \qquad j = 1, 2, ..., m-1,$

where $\Psi(t), I_n$ and $D_y f$ are the $n \times n$ transition matrix, identity matrix and Jacobian matrix, respectively. Since the solutions of the initial value problems (4.5) - (4.6) can be obtained by linear combinations of solutions from fundamental systems, the error functions (4.3) approximately equal the linear combinations of the columns of the solutions of the problems (4.7) - (4.8). Consequently, if we denote the solutions of the problems (4.7) - (4.8) by ${}^k\psi_j(t)$ we get

(4.9) $\quad {}^k\eta_j(t) \approx {}^k\psi_j(t){}^k v_j \qquad t \in [t_j, t_{j+1}],$
$\qquad j = 1, 2, ..., m-1.$

where the unknown vector coefficients ${}^k v_j$ approximately equal the values of ${}^k\eta_j(t_j)$ according to the substitutions of t_j for t in (4.9). So (4.9) can be replaced by

(4.10) $\quad y^*(t) \approx {}^k y_j(t) + {}^k\Psi_j(t){}^k\eta_j(t_j), \quad t \in [t_j, t_{j+1}],$
j=1,2,...,m-1.

In order to determine the approximate values of ${}^k\eta_j(t_j)$ we solvethe system of linear equations (4.11) which results, using the approximations (4.10), from the boundary conditions (3.4)-(3.5) and continuity conditions $y^*(t_j^-) = y^*(t_j^+)$ at the chosen points:

$$A^{*k}\eta_1(t_1) = c^* - A^{*k}y_1(t_1),$$

$$B^{*k}\Psi_{m-1}(t_m)^k\eta_{m-1}(t_{m-1}) = d^* - B^{*k}y_{m-1}(t_m),$$

$${}^k\Psi_{j-1}(t_j)^k\eta_{j-1}(t_{j-1}) - {}^k\eta_j(t_j) = {}^ky_j(t_j) - {}^ky_{j-1}(t_j) \; j \neq 1, m$$

at points t_j of continuity,

(4.11)

$$A_j^{-\,k}\Psi_{j-1}(t_j)^k\eta_{j-1}(t_{j-1}) + A_j^{+\,k}\eta_j(t_j) =$$

$$= s_j - A_j^{-\,k}y_{j-1}(t_j) - A_j^{+\,k}y_j(t_j)$$

at points t_j of discontinuity.

From the solution of the system above and according to (4.10) we obtain the new initial values in (4.2) for the (k+1) -st step of the algorithm:

$${}^{k+1}y(t_j^+) := {}^ky(t_j^+) + {}^k\eta_j(t_j), \qquad j = 1, 2, ..., m-1.$$

At the end of each iterative step we check the accuracy of the approximations using prescribed (mixed absolute/relative) error tolerances for the fulfilment of the boundary conditions (3.4) - (3.5) and the continuity conditions at the chosen points t_j. When the tolerances are met we finish the computations.

5. Numerical examples and relults

Numerical calculations have been performed for a telecomunication cable containing concentric aluminium wire screen ($R_{dc} = 0.493\Omega/km$) and double steel tape (30X0.5mm) armour. The measured and spline interpolated values of the outer surface impedance of the sheath are shown in Fig.2.

Fig.2. Resistance R_k and reactance $X_k(\Omega/km)$ of the surface impedance versus the current (A)

Concerning the earthing of the sheath a distributed uniform leakage with conductance G, and discrete earthing at both ends with identical resistance ha been assumed. In the parametric study, for the earthing conditions of the sheath the following three cases were considered:

Case no	Conductance G S/km	Earthing resistance $R_a = R_b$ Ω
1	0.1	100
2	2	5
3	10	0

Case no.1 results in a very weak earthing while no.3 realizes practically a perfect earthing of the sheath.

When specifying the induced e.m.f. a parallel exposure between an electric traction line and the telecommunication cable was assumed with a uniform mutual impedance of 0.267 Ω/km, phase 79.4 degree, at 50Hz. For the inducing current of traction line three versions were considered: uniform current produced by simple feeding system and lenght varying current of an autotransformer feeding system under normal operation and short circuit condition. The e.m.f. are listed in Tab.1. for the three cases considered.

Table 1. Versions of induced e.m.f. considered

Section length coordinate	Induction by traction line with simple feeding system		autotransformer system under normal operation		short circuit	
km	modulus V/km	phase deg.	modulus V/km	phase deg.	modulus V/km	phase deg.
0-1			16.84	98.9°	62.39	98.9°
1-2			25.15	90.5°	93.16	90.5°
2-3	96.39	85.9°	29.15	84.7°	108.0	84.7°
3-4			30.32	81.4°	112.3	81.4°
4-5			29.38	80.6°	108.8	80.6°
Total [V]	482	85.9°	130.1	85.9°	482	85.9°

Results for a selected version of parameters are listed in - Tab.2., where the screening factor k and the nominal screening factor k_N are defined by (2.4a) and (2.4b), respectively. For the comparison of different versions the sheath current (Fig.3) and the sheath-to-earth voltage (Fig.4) functions of length are presented. In addition, the components R_k and X_k (Fig.5) of the surface impedance have been also plotted together with the nominal screening factor k_N (Fig.6.).

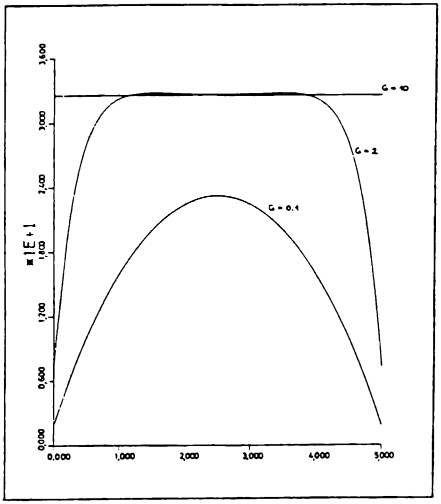
To interpret the effect of earthing conditions of the sheath on the sreening action, results for uniform e.m.f. are worth examining at first. The condition G=10 S/km and zero earthing resistance at both ends results in perfect earthing, thus the sheath current is uniform and the sheath-to-earth voltage is zero at any location. As a consequence of the uniform current the excitation of the sheath is uniform as well, therefore there is no variation in R_k and X_k, thus the screening factor k_N shows a uniform value as well. Considering the average earthing conditions occuring in the most practical cases, i.e. G=2 S/km and 5Ω earthing impedance, it can be seen that along the middle section, i.e. between 1.5 - 3.5

km, the sheath seems to be perfectly earthed, while the quantities vary at the vicinity of the line ends which are the so called end-effect zones. In the case of weak earthing, i.e. G=0.1 S/km and 100Ω earthing, the whole length is composed of two end-effect zones resulting in continuous variations in the quantities under study.

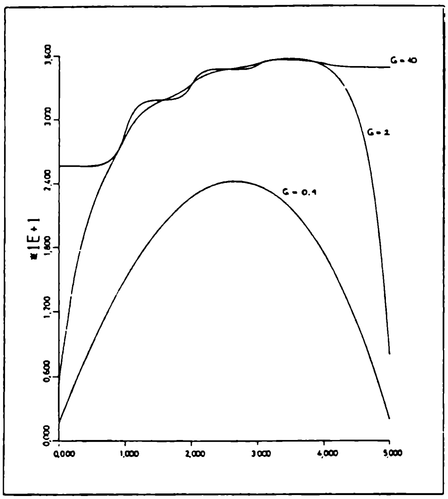
Concerning the results for length varying induced e.m.f., when G=10 S/km, the sheath current follows more or less strictly the variation in the e.m.f.; when G=2 S/km, the magnitude of current in the middle of the line follows the average e.m.f. levels of part sections; while when G=0.1 S/km, the current curve does not follow the local variations in e.m.f. at all. The consequences of these current variations can be seen in all of the studied quantities.

Table 2.
Results for uniform e.m.f., $G = 2S/km$, $R_a = R_b = 5\Omega$

	E.M.F.		VOLTAGE		CURRENT		RK	XK	KN		K	
KM	MODULUS	PHASE	MODULUS	PHASE	MODULUS	PHASE			MODULUS	PHASE	MODULUS	PHASE
0.0	9.639E+01	85.9	3.678E+01	238.2	7.356E+00	238.2	0.620	0.647	3.527E-01	298.7	3.762E-02	152.2
0.2	9.639E+01	85.9	2.275E+01	225.4	1.908E+01	234.7	0.955	1.069	2.554E-01	301.4	9.757E-02	148.8
0.4	9.639E+01	85.9	1.322E+01	209.7	2.589E+01	230.4	1.405	1.384	2.018E-01	306.6	1.324E-01	144.4
0.6	9.639E+01	85.9	7.742E+00	192.5	2.953E+01	226.6	1.690	1.541	1.797E-01	309.4	1.510E-01	140.7
0.8	9.639E+01	85.9	4.709E+00	175.5	3.137E+01	223.6	1.822	1.611	1.711E-01	310.5	1.605E-01	137.7
1.0	9.639E+01	85.9	2.966E+00	159.8	3.224E+01	221.4	1.880	1.642	1.676E-01	311.0	1.649E-01	135.5
1.0	9.639E+01	85.9	2.966E+00	159.8	3.224E+01	221.4	1.880	1.642	1.676E-01	311.0	1.649E-01	135.5
1.2	9.639E+01	85.9	1.907E+00	146.1	3.261E+01	219.9	1.904	1.654	1.662E-01	311.2	1.668E-01	133.9
1.4	9.639E+01	85.9	1.231E+00	134.5	3.273E+01	218.8	1.911	1.658	1.657E-01	311.2	1.674E-01	132.9
1.6	9.639E+01	85.9	7.865E-01	124.8	3.274E+01	218.1	1.912	1.658	1.657E-01	311.2	1.675E-01	132.2
1.8	9.639E+01	85.9	4.909E-01	116.8	3.271E+01	217.7	1.910	1.657	1.658E-01	311.2	1.673E-01	131.8
2.0	9.639E+01	85.9	2.920E-01	110.4	3.267E+01	217.4	1.908	1.656	1.659E-01	311.2	1.671E-01	131.5
2.0	9.639E+01	85.9	2.920E-01	110.4	3.267E+01	217.4	1.908	1.656	1.659E-01	311.2	1.671E-01	131.5
2.2	9.639E+01	85.9	1.534E-01	105.8	3.264E+01	217.3	1.906	1.655	1.660E-01	311.2	1.670E-01	131.3
2.4	9.639E+01	85.9	4.765E-02	103.4	3.263E+01	217.2	1.905	1.655	1.661E-01	311.2	1.669E-01	131.3
2.5	9.639E+01	85.9	2.302E-07		3.263E+01	217.2	1.905	1.654	1.661E-01	311.2	1.669E-01	131.3
2.6	9.639E+01	85.9	4.765E-02	283.4	3.263E+01	217.2	1.905	1.655	1.661E-01	311.2	1.669E-01	131.3
2.8	9.639E+01	85.9	1.534E-01	285.8	3.264E+01	217.3	1.906	1.655	1.660E-01	311.2	1.670E-01	131.3
3.0	9.639E+01	85.9	2.920E-01	290.4	3.267E+01	217.4	1.908	1.656	1.659E-01	311.2	1.671E-01	131.5
3.0	9.639E+01	85.9	2.920E-01	290.4	3.267E+01	217.4	1.908	1.656	1.659E-01	311.2	1.671E-01	131.5
3.2	9.639E+01	85.9	4.909E-01	296.8	3.271E+01	217.7	1.910	1.657	1.658E-01	311.2	1.673E-01	131.8
3.4	9.639E+01	85.9	7.865E-01	304.8	3.274E+01	218.1	1.912	1.658	1.657E-01	311.2	1.675E-01	132.2
3.6	9.639E+01	85.9	1.231E+00	314.5	3.273E+01	218.8	1.911	1.658	1.657E-01	311.2	1.674E-01	132.9
3.8	9.639E+01	85.9	1.907E+00	326.1	3.261E+01	219.9	1.904	1.654	1.662E-01	311.2	1.668E-01	133.9
4.0	9.639E+01	85.9	2.966E+00	339.8	3.224E+01	221.4	1.880	1.642	1.676E-01	311.0	1.649E-01	135.5
4.0	9.639E+01	85.9	2.966E+00	339.8	3.224E+01	221.4	1.880	1.642	1.676E-01	311.0	1.649E-01	135.5
4.2	9.639E+01	85.9	4.709E+00	355.5	3.137E+01	223.6	1.822	1.611	1.711E-01	310.5	1.605E-01	137.7
4.4	9.639E+01	85.9	7.742E+00	12.5	2.953E+01	226.6	1.690	1.541	1.797E-01	309.4	1.510E-01	140.7
4.6	9.639E+01	85.9	1.322E+01	29.7	2.589E+01	230.4	1.405	1.384	2.018E-01	306.6	1.324E-01	144.4
4.8	9.639E+01	85.9	2.275E+01	45.4	1.908E+01	234.7	0.955	1.069	2.554E-01	301.4	9.757E-02	148.8
5.0	9.639E+01	85.9	3.678E+01	58.2	7.356E+00	238.2	0.620	0.647	3.527E-01	298.7	3.762E-02	152.2

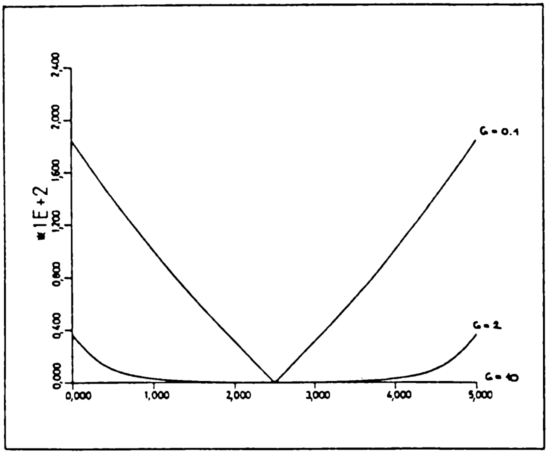


a. uniform induction

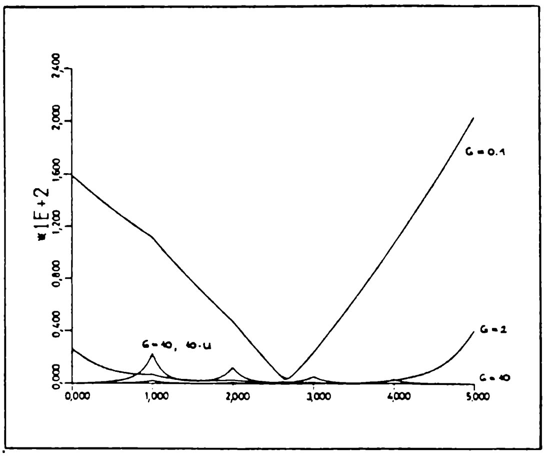


b. step-like induction by short circuit current

Fig.3. Sheath current (A) versus the length (km)

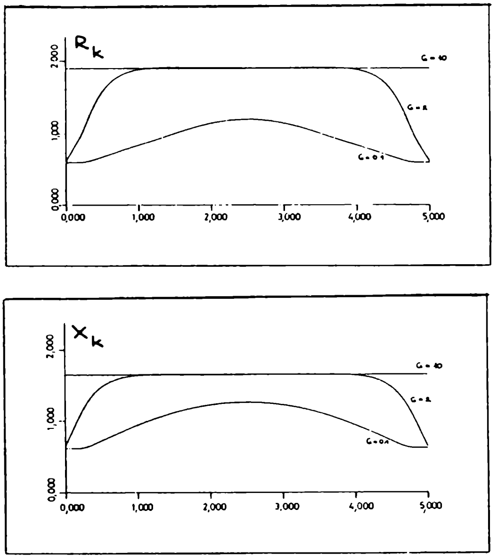


a. uniform induction

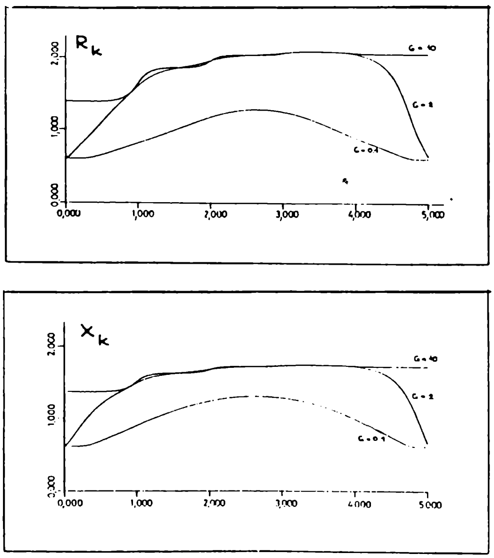


b. step-like induction by short circuit current

Fig.4. Sheath-to-earth voltage (V) versus the length (km)

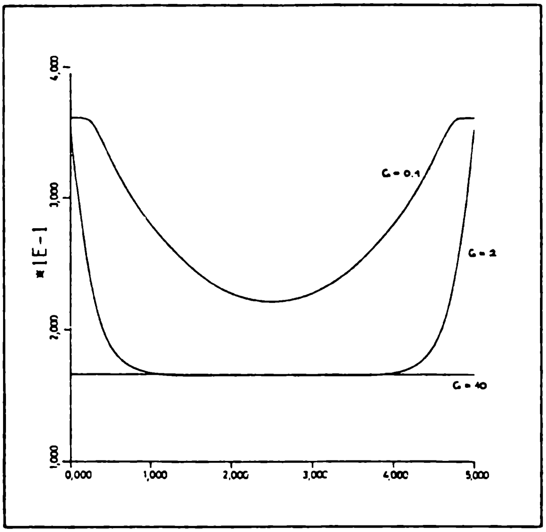


a. uniform induction

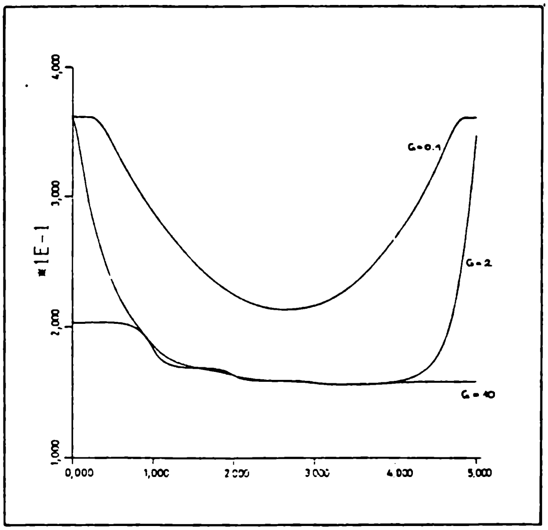


b. step-like induction by short circuit current

Fig.5. Resistance R_k and reactance $X_k (\Omega/km)$ of the surface impedance versus the length (km)

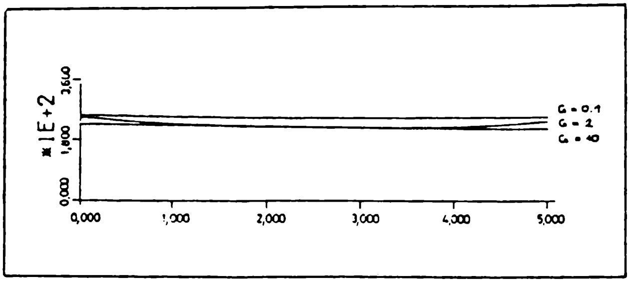


a. uniform induction

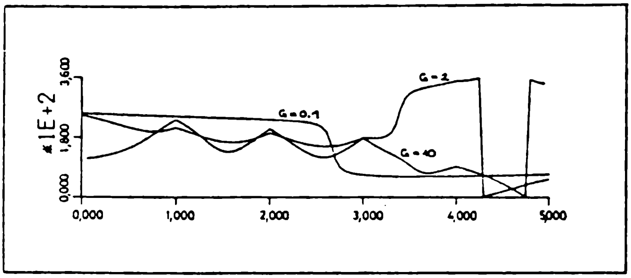


b. step-like induction by short circuit current

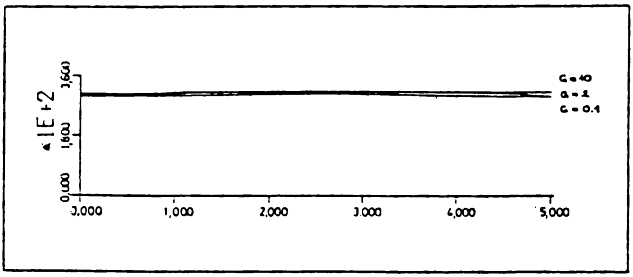
Fig.6. Screening factor k_N versus the length (km)



a. phase of current



b. phase of voltage



c. phase of screening factor

Fig.7. Variation of phase angle versus the length (in the case of step-like induction by short circuit current)

Concerning the phase angles (Fig.7), in the case of the current and screening factor they vary in a narrow range. The phase of voltage shows a significant (about 180^0) variation at the location of voltage minimum when G=0.1 or 2. When G=10, the phase of voltage has no importance due to the practically zero voltage magnitude.

The presented examples were calculated (in PL/1) on the computer R40 of the Computer Center of Eötvös Loránd University, Budapest.

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