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ON FUZZIFIED LINEAR PROGRAMMING PROBLEMS

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Abstract: In this paper a fuzzified linear programming problem is discussed, where the parameters are linear fuzzy functions.The maximum and the inequalities are considered in the sense of Goetschel-Voxman relation [1].

1.Introduction

In practical linear programming problems the parameters of the object and the constrains functions are often not well defined. This fact induces us to fuzzify these parameters i.e. to use fuzzy functions instead of real-valued parameters.

2.Basic notations and definitions

In this paper we adopt the notations and definitions used in [1]. We will say that $\mu : \mathbf{R} \to I = [0,1]$ is a fuzzy number if

(i) μ is upper semicontinuous, (ii) there are real numbers a, b, c and d, $c \le a \le b \le d$, such that $\mu(x) = 0$ outside $[c,d]$ and $\mu(x) = 1$ for each $x \in [a,b]$, furthermore μ is increasing in $[c,a]$ and decreasing in $[b,d]$.

The set of fuzzy numbers will be denoted by $\mathbf{F}$. Let $\mu \in \mathbf{F}$ and define $C_r(\mu)$ by

$$C_r(\mu) = \begin{cases} \{x \in \mathbf{R} \mid \mu(x) \ge r\} & \text{if } 0 < r \le 1 \\ \mathrm{cl}(\mathrm{supp}\ \mu) & \text{if } r = 0 \end{cases}$$

where cl(suppμ) denotes the closure of support of μ. We can identify a fuzzy number μ with the parametrical triplet

$$\{(a(r),\quad b(r),\ r) \mid r\ \in I\} \tag{2.1}$$

where $a(r)$ and $b(r)$ are the left and right endpoints of $C_r(\mu)$, respectively.

Let μ, $\theta \in \mathbf{F}$ be represented by (2.1) and $\{(c(r),\ d(r),\ r) \mid r \in I\}$,respectively. Suppose further that p is a real number. We can define [1]

- multiplication by scalar $p\mu$ by

$$p\mu := \begin{cases} \{(pa(r),\quad pb(r),\ r) \mid r \in I\} & \text{if } p \geq 0 \\ \{(pb(r),\quad pa(r),\ r) \mid r \in I\} & \text{if } p < 0 \end{cases}$$

- addition $\mu + \theta$ by

$$\mu + \theta := \{(a(r) + c(r),\ b(r) + d(r),\ r) \mid r \in I\};$$

- ordering $\mu\tilde{R}\theta$ generated by the relation R / $\leq$,$<$,$=$,$\geq$ or $>$ / on the real line :

$$\mu\tilde{R}\theta \text{ if } \left[\int_0^1 r(a(r) + b(r))dr\right] R \left[\int_0^1 r(c(r) + d(r))dr\right];$$

-function $S\ :\ \mathbf{F} \to \mathbf{R}$ by

$$S(\mu) := \int_0^1 r(a(r) + b(r))dr.$$

It is clear that

$$\mu\tilde{R}\theta \quad \text{iff} \quad S(\mu)RS(\theta).$$

A function $f : \mathbf{R} \to \mathbf{F}$ is called a fuzzy function. For each fuzzy function $f : \mathbf{R} \to \mathbf{F}$ let

$$\{(a(r,x),\ b(r,x),\ r) \mid r \in I\} \tag{2.2}$$

denote the representation of the fuzzy number $f(x)$ for each $x \in \mathbf{R}$

3. Main results

Let us consider the following special LP problem :

$$\max \sum_{j=1}^{n} c_j x_j \tag{3.1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i = 1, \ldots, m \tag{3.2}$$
$$x_j \geq 0, \quad j = 1, \ldots, n$$

A fuzzification of the LP problem (3.1), (3.2) may be defined as follows (other approaches see [2,3,4]). Let a fuzzy function $f : \mathbf{R} \to \mathbf{F}$ be given and instead of the parameters c_j, a_{ij}, b_j formulate the problem with the fuzzy number parameters $f(c_j)$, $f(a_{ij})$, $f(b_i)$:

$$\tilde{\max} \sum_{j=1}^{n} f(c_j) x_j \tag{3.3}$$

subject to

$$\sum_{j=1}^{n} f(a_{ij}) x_j \tilde{\leq} f(b_i), \quad i = 1, \ldots, m \tag{3.4}$$
$$x_j \geq 0, \quad j = 1, \ldots, n$$

where $\tilde{\max}$ is also regarded in the sense the ordering $\tilde{\leq}$. The membership functions of the fuzzy numbers $f(c_j), f(a_{ij}), f(b_i)$ can be considered as a possibility distribution of the corresponding parameters c_j, a_{ij}, b_i.

In the following a special class of fuzzy functions will be used to fuzzify the classical LP problem.

DEFINITION 3.1 A function $f : \mathbf{R} \to \mathbf{F}$ is called a homogeneous linear fuzzy function if

$$f(t) = t\tau, \qquad t \in \mathbf{R} \tag{3.5}$$

where $\tau \in \mathbf{F}$ is a fuzzy number. From the definition 3.1 it follows that $f(1) = \tau$. In the following theorem we will see that the fuzzified LP problem (3.3), (3.4) can be transformed into a special equivalent nonfuzzy LP problems if f is a homogeneous linear fuzzy function. The theorem in question can be stated as follows:

Theorem 3.1 *Let f be homogeneous linear fuzzy functions of the form (3.5) then the solution of the fuzzy LP problem (3.3),(3.4) is:*

(i) *the solution of the classical LP problem*

$$\min \sum_{j=1}^{n} c_j x_j$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \qquad i = 1, \dots, m$$

$$x_j \geq 0, j = 1, \dots, n$$

if $\tau \tilde{<} \tilde{0}$;

(ii) *all* $x_j \geq 0, \quad j = 1, \dots, n,$ if $\tau \tilde{=} \tilde{0}$;

(iii) *the classical LP problem*

$$\max \sum_{j=1}^{n} c_j x_j$$

subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \qquad i = 1, \ldots, m$$

$$x_j \geq 0, j = 1, \ldots, n$$

if $\tau \tilde{>} \tilde{0}$;
where $\tilde{0} \in F$ *is represented parametrically by* $\{(0, 0, r) | r \in I\}$.

Proof. From the definition 3.1 we have

$$\sum_{j=1}^{n} f(a_{ij})x_j = f(1)\sum_{j=1}^{n} a_{ij}x_j = \tau \sum_{j=1}^{n} a_{ij}x_j.$$

Consequently, the constrains (3.4) will be fulfilled if

$$\left[\sum_{j=1}^{n} a_{ij}x_j\right] S(\tau) \leq b_i S(\tau) \tag{3.6}$$

Furthermore x^* is a solution of (3.3), (3.4) if the inequality

$$\left[\sum_{j=1}^{n} c_j x_j\right] S(\tau) \leq \left[\sum_{j=1}^{n} c_j x_j^*\right] S(\tau) \tag{3.7}$$

holds true for all $x_j \geq 0$, $j = 1, \ldots, n$, satisfying (3.7). From (3.6) if follows that the nonfuzzy contraints set is one of the following sets:

$$C_1 = \{x \in \mathbf{R}^n | x \geq 0, \quad \sum_{j=1}^{n} a_{ij}x_j \leq b_i, \ i = 1, \ldots, m\}, \quad \text{if } \tau \tilde{>} \tilde{0},$$

$$C_2 = \{x \in \mathbf{R}^n | x \geq 0\}, \quad \text{if } \tau \tilde{=} \tilde{0},$$

$$C_3 = \{x \in \mathbf{R}^n | x \geq 0, \quad \sum_{j=1}^{n} a_{ij}x_j \geq b_i \ i = 1, \ldots, m\}, \quad \text{if } \tau \tilde{<} \tilde{0},$$

and under these constans the optimality condition (3.7) is

$$\sum_{j=1}^{n} c_j x_j \leq \sum_{j=1}^{n} c_j x_j^* \quad \text{for all } x \in C_1 \text{ if } \tau \tilde{>} \tilde{0},$$

$$\sum_{j=1}^{n} c_j x_j \geq \sum_{j=1}^{n} c_j x_j^* \quad \text{for all } x \in C_2 \text{ if } \tau \tilde{<} \tilde{0},$$

(3.7) is satisfied for all $x \in C_3$ if $\tau \tilde{=} \tilde{0}$.

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