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SOME ASPECTS OF LAGUERRE FILTERING USAGE FOR TIME SERIES ANALYSIS

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Abstract. This paper discusses using a discrete Laguerre filter for some time series problems in the frequency domain. The Laguerre transformation of an ARMA time series model is considered. The $\beta-$function is considered and the possible application of FFT for efficient computation is pointed out.

1. Introduction

The application of orthogonal polynomials in both deterministic and stochastic systems look back over a long time. System identification based on frequency-domain interpretation of discrete-time signals plays a significant role in the control theory and design. Estimating the poles associated with input and output signals as well as the transfer functions is an efficient approach to identifying the system dynamics; knowing – exactly or approximately — the location of system poles is sufficient in estimating the whole system dynamics by applying the principles of representations in orthogonal rational bases, [12], [4].

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In the paper, [7] a new method was proposed that efficiently identifies the poles in a linear system from frequency domain data. The discrete rational transfer function was represented in rational Laguerre–basis, where powers of the Blaschke–function can express the basis elements. This function can be interpreted as a congruence transform on the Poincaré unit disc model of the hyperbolic geometry, leading to a nice geometric interpretation of the identification algorithm. The reconstruction of a pole can be obtained as a hyperbolic transform of the limit of a sequence formed of quotients of the Laguerre–Fourier coefficients of the function. Using an efficient FFT-based algorithm, the Laguerre–Fourier coefficients can be estimated from frequency domain data.

The pole–identification algorithm introduced in [7] uses frequency–domain data that can be either direct frequency–domain measurements or spectral estimations based on discrete time–domain samples, see also [8]. Moreover, the phase function (β–function) of the Blaschke product was introduced in [7] as well, and it has turned out very useful in calculating periodograms and other frequency domain methods connected to Laguerre filters. The proposed pole-identification method requires non-uniformly spaced frequency sample points which are determined by this β–function. This allows the standard FFT algorithm to estimate the Laguerre–Fourier coefficients. The poles are identified from multiple Laguerre–series coefficients corresponding to different Laguerre parameters, thus multiple measurement sequences are required on non-uniformly spaced frequency scales. First, we consider an ARMA time series model with the stochastic spectral representation. The discrete Laguerre is applied in the frequency domain. The connection between the series in time and the Laguerre transformed series is given by a linear fractional (bilinear) transformation which is an all-pass filter. We apply the Laguerre shift for the construction of the ZAR model. In Section 3 the $\beta-$function is considered using FFT to apply the Laguerre method.

In this paper, we concentrate on the ideas rather than the general treatment therefore we shall focus on Laguerre polynomials although similar results can be deduced for Kautz and generalized orthonormal rational basis functions, [6], [10] as well. Throughout this paper, we shall use notations which are custom in the system theory instead of traditional time series notations.

Laguerre polynomials are widely known for forming a closed orthogonal system over $\mathbb{L}_2(\mathbb{R}_+)$, [11]. The discrete counterpart is called the discrete Laguerre-type functions due to Gottlieb, [3]. Its z-transform, the discrete Laguerre filter, [5], became a basic tool for system theory. This latter is defined as follows. Let $|\alpha| < 1$ and define the Blaschke factor with poles that corresponds to α

$$B_k(z,\alpha) = \left(\frac{1-\alpha z}{z-\alpha}\right)^k .$$

The Laguerre base functions are defined by

$$L_k(z,\alpha) = \mathfrak{m}_\alpha(z)\, B_k(z,\alpha) = \frac{\sqrt{1-\alpha^2}}{z-\alpha}\left(\frac{1-\alpha z}{z-\alpha}\right)^{k-1},$$

for $k = 1, 2, \ldots n$, where

$$\mathfrak{m}_\alpha(z) = \frac{\sqrt{1-|\alpha|^2}}{z-\alpha}.$$

2. Discrete Laguerre filtered time series

We shall consider a stationary time series

$$X_t = \int_{-\pi}^{\pi} e^{\mathrm{i}\omega t} G_X\left(e^{\mathrm{i}\omega}\right) W(d\omega), \tag{2.1}$$

with transfer function $G_X(z)$ and stochastic spectral measure $W(d\omega)$ such that $\mathrm{E}\, W(d\omega) = 0$ and $\mathrm{E}\,|W(d\omega)|^2 = \sigma^2 d\omega/2\pi$. Assume that the transfer function G_X is rational, i.e. $G_X(z) = B(z)/A(z)$ where the polynomials are written in the following form $B(z) = z^{n_a-1} + b_1 z^{n_a-2} + \cdots + b_{n_b} z^{n_a-n_b}$, $n_a - n_b \geq 0$, $b_{n_b} \neq 0$, and $A(z) = z^{n_a} + a_1 z^{n_a-1} + \cdots + a_{n_a}$, $a_{n_a} \neq 0$. In this case X_t is also called ARMA(n_a,n_b) model. The necessary and sufficient conditions of stationarity of X_t is that $A(z)$ has no roots on unite circle. Moreover if all the roots of $A(z)$ are inside of the unite circle then X_t is causal

$$X_t = \sum_{k=1}^{\infty} g_k W_{t-k} = G_X(q)\, W_t,$$

where

$$W_t = \int_{-\pi}^{\pi} e^{\mathrm{i}\omega t} W(d\omega),$$

is a white noise series, $q^{-k} W_t = W_{t-k}$. The spectral density of X_t is $\Phi_X(\omega) = \sigma^2 \left|G_X\left(e^{\mathrm{i}\omega}\right)\right|^2$, and the covariance function of X_t is

$$C_X(\tau) = \mathrm{Cov}\left(X_{t+\tau}, X_t\right) = \int_{-\pi}^{\pi} e^{\mathrm{i}\omega\tau} \left|G\left(e^{i\omega}\right)\right|^2 \frac{\sigma^2 d\omega}{2\pi}.$$

Note here that in time series analysis polynomials $A(z)$ and $B(z)$ traditionally have different forms, for instance, the denominator writes as

$1+a_1z+\cdots+a_{n_a}z^{n_a}$, with the straightforward consequence that the assumption of causality of X_t is that all the roots of $A(z)$ be outside of the unite circle.

Let us apply the Laguerre filter L_k on X_t and get

$$\widetilde{X}_k(t,\alpha)=\int_{-\pi}^{\pi}e^{\mathrm{i}\omega t}L_k\left(e^{\mathrm{i}\omega},\alpha\right)G_X\left(e^{\mathrm{i}\omega}\right)W(d\omega)\,,\quad k=0,1\ldots. \tag{2.2}$$

We call attention to the notation here and from now on the subscript k of series $\widetilde{X}$ denotes the order of Laguerre shift but time.

The covariance

$$\begin{aligned}&\operatorname{Cov}\left(\widetilde{X}_k(t,\alpha),\widetilde{X}_{k+m}(t,\alpha)\right)=\\ &=\int_{-\pi}^{\pi}L_k\left(e^{\mathrm{i}\omega},\alpha\right)L_{k+m}\left(e^{-\mathrm{i}\omega},\alpha\right)\left|G_X\left(e^{\mathrm{i}\omega}\right)\right|^2\frac{\sigma^2d\omega}{2\pi}=\\ &=\left(1-\alpha^2\right)\frac{\sigma^2}{2\pi\mathrm{i}}\oint\frac{1}{(z-\alpha)(1-\alpha z)}\left(\frac{1-\alpha z}{z-\alpha}\right)^m\left|G_X(z)\right|^2dz\end{aligned} \tag{2.3}$$

shows that for any fixed $t=t_0$, say, $\widetilde{X}_k(t_0,\alpha)$ is stationary. Let us put $t_0=0$ and denote $\widetilde{X}_k(0,\alpha)=\widetilde{X}_k(\alpha)$. The spectral representation of the $\widetilde{X}_k(\alpha)$ follows from (2.2) as follows. Let us introduce the change of variables

$$e^{\mathrm{i}\varpi}=\frac{e^{\mathrm{i}\omega}-\alpha}{1-\alpha e^{\mathrm{i}\omega}}, \tag{2.4}$$

with the Jacobian

$$d\omega=\frac{1-\alpha^2}{\left|e^{\mathrm{i}\varpi}-\alpha\right|^2}d\varpi=\left|m_\alpha\left(e^{\mathrm{i}\varpi}\right)\right|^2d\varpi, \tag{2.5}$$

see [4, p. 77]. Hence the stochastic spectral measure transforms to

$$W(d\omega)=\frac{\sqrt{1-\alpha^2}}{e^{\mathrm{i}\varpi}-\alpha}W(d\varpi)\,, \tag{2.6}$$

and $\mathrm{E}\left|W\left(d\varpi\right)\right|^{2}=\sigma^{2}d\varpi/2\pi$. Therefore we have

$$\begin{aligned}\widetilde{X}_{k}\left(\alpha\right) &= \int\limits_{-\pi}^{\pi}\sqrt{1-\alpha^{2}}\frac{e^{\mathrm{i}\varpi}+\alpha}{1-\alpha^{2}}e^{\mathrm{i}\varpi k}G_{X}\left(\frac{e^{\mathrm{i}\varpi}+\alpha}{1+\alpha e^{\mathrm{i}\varpi}}\right)\frac{\sqrt{1-\alpha^{2}}}{e^{\mathrm{i}\varpi}+\alpha}W\left(d\varpi\right)= \\ &= \int\limits_{-\pi}^{\pi}e^{\mathrm{i}\varpi k}G_{X}\left(\frac{e^{\mathrm{i}\varpi}+\alpha}{1+\alpha e^{\mathrm{i}\varpi}}\right)W\left(d\varpi\right)= \\ &= \int\limits_{-\pi}^{\pi}e^{\mathrm{i}\varpi k}G_{\widetilde{X}}\left(e^{\mathrm{i}\varpi}\right)W\left(d\varpi\right). \qquad (2.7)\end{aligned}$$

The transformation 2.4 corresponds to the first-order all-pass shift operator $B_{1}\left(z,\alpha\right)$. Now we consider the inverse transformation

$$\mathcal{B}_{1}\left(z,\alpha\right)=\frac{z+\alpha}{1+\alpha z}=\frac{1+\alpha z^{-1}}{z^{-1}+\alpha}$$

of $B_{1}\left(z,\alpha\right)$ and define the corresponding base function by

$$\mathcal{L}_{t}\left(z,\alpha\right)=\frac{\sqrt{1-\alpha^{2}}}{1+\alpha z}\left(\frac{z+\alpha}{1+\alpha z}\right)^{t-1}=\mathcal{M}_{\alpha}\left(z\right)\mathcal{B}_{t-1}\left(z,\alpha\right),$$

where

$$\mathcal{M}_{\alpha}\left(z\right)=\frac{\sqrt{1-\alpha^{2}}}{z+\alpha}$$

is the normalizing term. If we are given the series $\widetilde{X}_{k}\left(\alpha\right)$ by (2.7) then put $k=0$ and apply the transformation $\mathcal{L}_{t}$

$$Y_{t}\left(\alpha\right)=\int\limits_{-\pi}^{\pi}\mathcal{L}_{t}\left(z,\alpha\right)G_{\widetilde{X}}\left(e^{\mathrm{i}\varpi}\right)W\left(d\varpi\right).$$

The transformation

$$e^{\mathrm{i}\omega}=\frac{e^{\mathrm{i}\varpi}+\alpha}{1+\alpha e^{\mathrm{i}\varpi}},$$

with Jacobians

$$\begin{aligned}d\varpi &= \frac{\alpha^{2}-1}{\left|1+\alpha e^{\mathrm{i}\omega}\right|^{2}}d\omega,\text{ and} \\ W\left(d\varpi\right) &= \frac{\sqrt{1-\alpha^{2}}}{e^{\mathrm{i}\varpi}+\alpha}W\left(d\omega\right),\end{aligned}$$

provides the time series

$$\begin{aligned} Y_t(\alpha) &= \int_{-\pi}^{\pi} G_{\widetilde{X}}\left(e^{\mathrm{i}\varpi}\right) \mathcal{L}_t\left(e^{\mathrm{i}\varpi}\right) W(d\varpi) = \\ &= \int_{-\pi}^{\pi} e^{\mathrm{i}\omega t} G_{\widetilde{X}}\left(\frac{e^{\mathrm{i}\omega}-\alpha}{1-\alpha e^{\mathrm{i}\omega}}\right) W(d\omega) = \\ &= \int_{-\pi}^{\pi} e^{\mathrm{i}\omega t} G_X\left(e^{\mathrm{i}\omega}\right) W(d\omega) = X_t. \end{aligned}$$

The result is the following

Lemma 2.1. *Assume the transfer function $G_X(z) = B(z)/A(z)$ of the series X_t, defined by (2.1), to be analytic in $|z| > 1$, continuous in $|z| \le 1$ and let $\widetilde{X}_k(\alpha)$ be the discrete Laguerre filtered series (2.2) of X_0. Then $\widetilde{X}_k(\alpha)$ is stationary with spectral representation*

$$\widetilde{X}_k(\alpha) = \int_{-\pi}^{\pi} e^{\mathrm{i}\varpi k} G_X\left(\frac{e^{\mathrm{i}\varpi}+\alpha}{1+\alpha e^{\mathrm{i}\varpi}}\right) W(d\varpi),$$

covariance function (2.3) and spectrum

$$\Phi_{\widetilde{X}}(\varpi) = \sigma^2 \left| G_X\left(\frac{e^{\mathrm{i}\varpi}+\alpha}{1+\alpha e^{\mathrm{i}\varpi}}\right)\right|^2.$$

Moreover the spectrum of X_t is given in terms of the transfer function of $\widetilde{X}_k(\alpha)$ by

$$\Phi_X(\omega) = \sigma^2 \left| G_{\widetilde{X}}\left(\frac{e^{\mathrm{i}\omega}-\alpha}{1-\alpha e^{\mathrm{i}\omega}}\right)\right|^2.$$

It is worth mentioning a model called ZAR ([14]) which generalizes the autoregressive structure for the Laguerre filtered series $\widetilde{X}_k(t,\alpha)$. Actually consider the linear prediction of X_t in terms of $\widetilde{X}_k(t,\alpha)$, $t = 1, 2, \ldots n$, then we arrive the equation

$$X_t + \gamma_1 \widetilde{X}_1(t,\alpha) + \gamma_2 \widetilde{X}_2(t,\alpha) + \cdots + \gamma_n \widetilde{X}_n(t,\alpha) = e_t. \tag{2.8}$$

If we notice that

$$L_k(z,\alpha) = L_{k-1}(z,\alpha)\frac{1-\alpha z}{z-\alpha}, \quad k = 2, \ldots$$

we can define the Laguerre shift $\widetilde{X}_k$ of X_t by $\widetilde{X}_k(t,\alpha) = Z_\alpha \widetilde{X}_{k-1}(t,\alpha)$, such that we apply the filter $(1-\alpha z)/(z-\alpha)$ on $\widetilde{X}_{k-1}$. The equation (2.8) is written as

$$X_t + \left(\gamma_1 Z_\alpha + \gamma_2 Z_\alpha^2 + \cdots + \gamma_n Z_\alpha^n\right) X_t = e_t.$$

This problem leads to linear regression where the matrix is calculated from the covariances of $\widetilde{X}_k$, see also [13] and [9]. Some statistical problems investigated by [13] would follow now from the Lemma 2.1. Observe the difference between an ordinary AR time series and a ZAR one where both the coefficients γ_k and the error term e_t depend not only on the order but from both α and n as well.

3. β–function

The computation of the spectrum for the series $\widetilde{X}_k$ is not so direct as for a series X_t that depends on time, [2], [1]. The idea uses transformations of the previous section in a particular way. We introduce the phase function of a single term $B_1(z,\alpha)$ which corresponds to the transformation (2.4)

$$B_1(z,\alpha) = \frac{1-\alpha z}{z-\alpha} = e^{\mathrm{i}\beta_\alpha(\omega)}.$$

Then the inverse transformation of the generalized shift operator Z_α is given by

$$\mathcal{B}_1(z,\alpha) = \frac{z+\alpha}{1+\alpha z}.$$

Now define the $\beta-$function and its inverse by the equations

$$e^{\mathrm{i}\varpi} = \frac{e^{\mathrm{i}\omega}-\alpha}{1-\alpha e^{\mathrm{i}\omega}} = e^{\mathrm{i}\beta_\alpha(\omega)}, \text{ and } e^{\mathrm{i}\omega} = \frac{e^{\mathrm{i}\varpi}+\alpha}{1+\alpha e^{\mathrm{i}\varpi}} = e^{\mathrm{i}\beta_\alpha^{-1}(\varpi)}.$$

We find that the frequency warp maps show up $\varpi = \beta_\alpha(\omega)$ and $\omega = \beta_\alpha^{-1}(\varpi)$ respectively. The $\beta-$function can also be clearly expressed by the formulae

$$\begin{aligned} \beta(\omega) &= 2\arctan(\mu\tan(\omega/2)), \text{ and} \\ \beta_\alpha^{-1}(\varpi) &= 2\arctan(\widetilde{\mu}\tan(\varpi/2)), \end{aligned}$$

where

$$\mu = \frac{1+\alpha}{1-\alpha}, \text{ and } \widetilde{\mu} = \frac{1-\alpha}{1+\alpha},$$

respectively. The transfer functions Φ_X and $\Phi_{\widetilde{X}}$ are given in terms of $\beta-$functions as

$$\Phi_X(\omega) = \Phi_{\widetilde{X}}(\beta_\alpha(\omega)), \text{ and} \tag{3.1}$$

$$\Phi_{\widetilde{X}}(\varpi) = \Phi_X\left(\beta_\alpha^{-1}(\varpi)\right). \tag{3.2}$$

Now we have

$$\operatorname{Cov}\left(\widetilde{X}_k\left(\alpha\right),\widetilde{X}_{k+m}\left(\alpha\right)\right)=\frac{\sigma^2}{2\pi}\int_{-\pi}^{\pi} e^{\mathrm{i}m\omega}\left|G_X\left(e^{\mathrm{i}\beta_\alpha^{-1}(\omega)}\right)\right|^2 d\omega,$$

i.e. the spectrum of $\widetilde{X}_k\left(\alpha\right)$ is $\sigma^2\left|G_X\left(e^{\mathrm{i}\beta_\alpha^{-1}(\omega)}\right)\right|^2$. The model parameters can be computed from a DFT of appropriately warped frequency response data. spectrum of X_t

$$\begin{aligned} C_X\left(\tau\right) &= \operatorname{Cov}\left(X_{t+\tau},X_t\right)=\int_{-\pi}^{\pi} e^{\mathrm{i}\omega\tau}\left|G_X\left(e^{i\omega}\right)\right|^2\frac{\sigma^2 d\omega}{2\pi} \\ &= \int_{-\pi}^{\pi} e^{\mathrm{i}\omega\tau}\left|G_{\widetilde{X}}\left(e^{\mathrm{i}\beta_\alpha(\omega)}\right)\right|^2\frac{\sigma^2 d\omega}{2\pi}. \end{aligned}$$

The $\beta-$transformations (3.1) and (3.2) makes possible the use of the computationally very efficient FFT for getting the Laguerre transfer function and as a consequence the Laguerre spectrum. The Fourier transform of the data provides the estimate of the spectrum at frequency $\beta_\alpha^{-1}\left(2\pi j/N\right)$, see Computational algorithms, [1], [8]. During these transformations one has to pay attention to the effect of the non-uniformly spaced frequency scale produced by the inverse argument-transform β_α^{-1}, see Figures 1 and 2.

These plots concern two basic issues of time series analysis, namely the assumption of stationarity and long-range dependence. The basic assumption of stationarity is that there should be no pole of the transfer function on the unit circle. Figure 1 shows that the $\beta-$transformation can conclude stationarity even if a pole is close to the unit circle.

Let us recall the notion of long-range dependence. The ARMA(n_a,n_b) time series X_t is long-range dependent (with long memory) if its transfer function is given by

$$G_X\left(z\right)=\frac{B\left(z\right)}{\left(1-z\right)^d A\left(z\right)},$$

where $d\in(0,1/2)$ and the polynomials are given in Section 1. In this case X_t is also called FARIMA(n_a,d,n_b) model. The spectrum of X_t now is

$$\Phi_X\left(e^{\mathrm{i}\omega}\right)=\sigma^2\left|1-e^{\mathrm{i}\omega}\right|^{-2d}\left|\frac{B\left(e^{\mathrm{i}\omega}\right)}{A\left(e^{\mathrm{i}\omega}\right)}\right|^2.$$

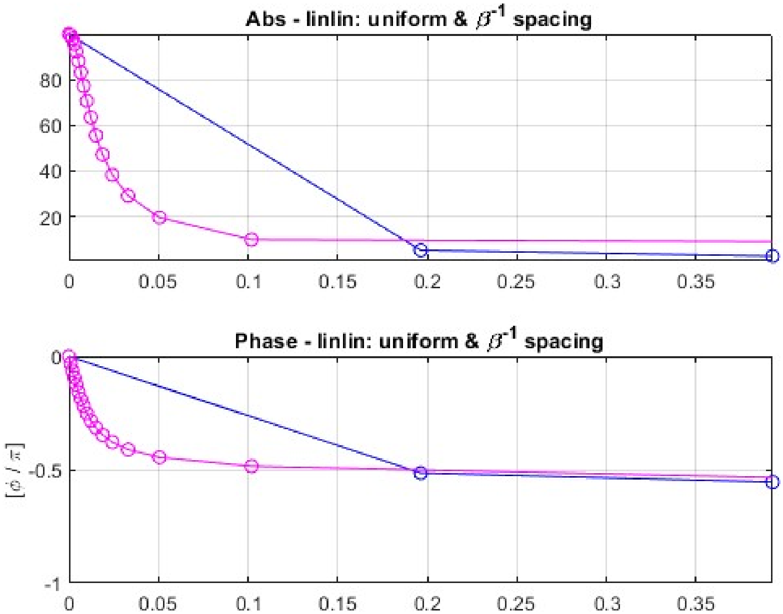

Figure 1. The approximation of the pole ($\omega = 0.99$) by FFT (blue circles) with linear frequency spacing and by using $\beta_\alpha^{-1}(\omega)$. While the FFT has no information between frequencies rad 0 and 0.2 the $\beta_\alpha^{-1}(\omega)$ provides a clear limit when getting closer to the pole.

It follows that the second order moments of X_t are finite and $\Phi_X\left(e^{\mathrm{i}\omega}\right) \sim |\omega|^{-2d}$ around zero. Note that in general a stationary time series X_t is long-range dependent iff there exists a real number $d \in (0, 1/2)$ and a constant $c_\Phi > 0$ such that

$$\lim_{\omega \to 0} \frac{\Phi_X(\omega)}{c_\Phi |\omega|^{2d}} = 1.$$

Figure 2 points to the linearity of the transformation β_α^{-1} around zero hence there is no extra information about long-range dependence contained in the Laguerre spectrum.

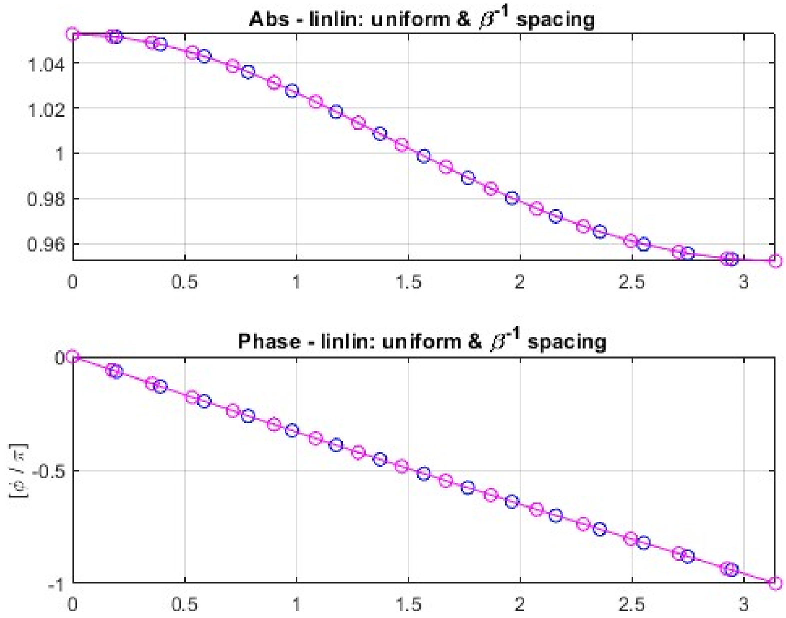

Figure 2. There is no significant difference between linear spacing and spacing by $\beta_{\alpha}^{-1}(\omega)$ when the frequency is close to zero, actually $\omega = 0.01$.

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