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CHESS AND BRIDGE CLUSTERING THE COUNTRIES

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Dedicated to the memory of Professor Antal Iványi

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Abstract. This paper investigates the two most popular brain games over the world: chess and bridge. The used data are the number of chess players beyond Elo 2000 and the number of bridge masterpoints earned by the countries over the world. We check the developments over the last 15 years and investigate the clusters. It turns out that there is some, but not too strong dependence between the level and the tendencies of the two sports within a country.

1. Introduction

In this dedicated paper we deal with the two brain games, both of which were the favourites of Professor Antal Iványi. He had a FIDE rating of over 2100 Elo points for several years and participated successfully on different bridge tournaments at various levels. On the top of this fact, one of the authors even participated together with Antal Iványi on some interesting combined chess-bridge events organised by the Hungarian federations of the two sports (see a short report on one of these tournaments – in Hungarian – in [1]).

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The challenge in this analysis is the fact that there are quite different measurement methods in these sports. Chess uses the famous Elo (In Hungarian "Élő") points - named after the Hungarian-born chess player and professor of Physics, Árpád Élő. This is a well-elaborated rating system. Here the earned points depend not only on the result itself, but on the strength of the opponents as well.

In competitive bridge luck plays a slightly more important role, thus the rating system here is based on a so-called masterpoint scheme, which does not depend on the strength of the opponents, but on the strength of the competition. Such masterpoints (MPs) can be earned at different levels - here we focus on the international level, which can be got at selected international tournaments (the rules can be found in [2]). The World Bridge Federation decides about the points allocated to the main international events, such as the World Team Championships, where each participant gets MPs - but the participation is based on a qualifying system, or World Youth Championships, where the first 8 teams get MPs. There is a separate list for Open, Women and Senior competitions. We restrict our analysis to the most general Open ones. On some of the main open tournaments a given percentage of the participants get MPs, based on their results. Thus these international MPs definitely honour the world class achievements, while in chess there is a much wider pool of players, who have Elo points. In this analysis we do not want to completely balance the achievements in these two sports, the used data corresponds to the much wider basis of chess.

The focus of this paper is to reveal the developments in the last 15 years especially to detect if there is any dependence between the achievements in these two sports. After some initial investigations we use cluster analysis for finding out the most important distinct groups. Our next step is to reveal the dependencies between the two clusterings.

First we introduce the data and the used models in Section 2. Next we show the applications of the models in Section 3. In Section 4 we give some conclusions.

2. Data and methods

2.1. Data preparation

There are completely different rating methods used in these sports. Chess ratings are administered by FIDE (Federation Internationale des Echecs, World

Chess Federation) and are published on a monthly basis. However there were several changes applied in the last decades – mainly in the data structure, but also in the frequency of its publication. Till 2010 the ratings were published quarterly, but since then there are monthly lists. We have used a half-year cycle: for each year two complete data sets were downloaded from the public site `https://ratings.fide.com`. The data size has also increased dramatically: from 3Mb in 2001 to 32Mb in 2016. This increase has two reasons: the most important change is that now FIDE administers all ratings, while it was used only for the international level players till 2007. The homogeneity of the Elo points of different time periods was investigated in [9], also by data mining tools – using chess-analysis computer programs, which are by now widely available. The results were encouraging, as these authors found no proof for inflation: the recent higher ratings are simply due to the fact that the actual players play chess better. The all-time high rate of the reigning world champion, Magnus Carlsen is 2882 – while Kasparov had "only" 2851 points in 1999 when he was at the top. However, still there is a debate about possible inflation - see [3] for an argument on this side. But even if this inflation indeed exists, its effect cannot be more than 30 points in 15 years, which is a rather negligible amount. And its distribution among countries is even more unclear, so we shall not deal with its existence.

We decided to include those players, who have an Elo rating above 2000. This is a level, which cannot be reached just by talent, one needs some kind of training and successful participation on official tournaments to achieve it. But, as we do not intend to focus on the very top, but on a much wider pool – where statistics can be applied with a higher power –, just the number of such players is used as the main source of information for the chess-level in a country.

For the bridge we have used the annual masterpoints earned by players from the given country. So here everyone starts from zero at the beginning of each year, not like in the MP player lists, where earlier earned MPs are also counted, but by an annual discount factor of 0.85. And as the number of bridge players as well as the number of such international competitions where MPs are given is much less than those of chess, here all of the original MP data is used.

As the size of the countries is obviously different, which has an obvious effect on the number of players, we got rid of the population differences by dividing our numbers by the population of the country. As a reliable source we have used the United Nation's downloadable file [4], for the years 2001, 2008 and 2015. The world map file together with the needed mapping tools are included in the `R` package `rworldmap`.

First we counted the number of chess players for each year and country and aggregated the bridge MPs similarly. Thus we got 15 observations per country for bridge and 31 for chess (as here we have two data sets per year).

For the bridge data there was a biannual period in the total number of MPs – obviously due to such events, which are held every two years only – but otherwise neither the sum of the MPs pro year, nor the countries which got MPs show any tendencies.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
41	45	49	74	40	43	38	74	36	54	52	66	38	50	45
79	103	74	111	82	99	94	115	62	178	91	102	94	222	103

Table 1. The first line: year (2000+) between 2001 and 2015; the second line: the number of countries with MPs, the last line: the number of annually given MPs (in thousands).

However, both the number of chess players and the number of countries with players above Elo 2000 show a significant upward trend in the investigated period. The increase in the number of countries with at least one player above 2000 and the increase in the total number of players is seen in Figure 1.

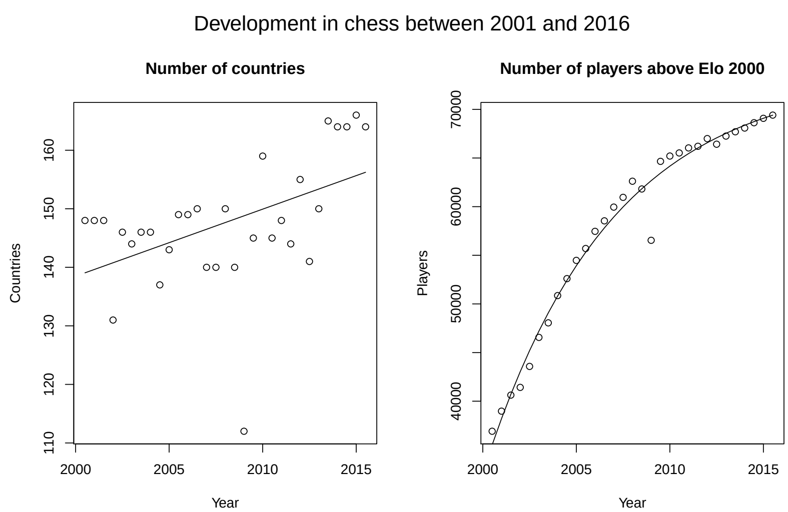

Figure 1. The increase in the number of countries with at least one chess player over Elo 2000 and the total number of chess players over Elo 2000, between 2001 and 2015 together with the best fitting line/curve

The left panel also shows a linear trend, estimated by the method of least squares. It has a slope of 0.57, so we can say that in the observed period a new country emerges in approximately every second year. The second panel shows the increase in the number of players, together with the fitted saturation curve of the form $a(1 - b\exp\{-cx\})$, known as the von Bertalanffy curve (see eg. [10]). It is an interesting result based on this model, that the estimated maximal number is $\hat{a} = 73632$. Based on the approximate normality of the estimator, the upper 95% quantile was estimated as 76144 - while a parametric bootstrap based on the residuals (see eg. Kreiss and Lahiri [8]) gave 76022 for the same amount - so we may be relatively confident in the error of the estimate (assuming that our model is correct).

The random fluctuations of the country level data were substantial – especially in the bridge data series – so first some smoothing methodologies were used. In this data preparation step in case of the chess data we have used a simple quadratic regression (with time and its square being the independent variables) - and we used the estimated values for the years 2001, 2008 and 2015. The values f_i for these three dates of this smoothed sequence ($i = 1, 2, 3$) were normalised by the countries' actual population, let us denote these quantities by an asterisk (*) over the original notation. As we were also interested in the tendencies, the ratios of the subsequent quantities have been calculated as well: $r_i := f^*_{i+1}/f^*_i$. In the case of chess, the clustering was based on the standardised 5-dimensional data $f^*_i/\mathrm{sd}(f^*_i)$ $i = 1, 2, 3$ and $r_j/\mathrm{sd}(r_j)$ $j = 1, 2$ (sd denotes the standard deviation throughout the paper).

For the bridge data, the regression has not produced reliable results. These data show enormous fluctuation, as here not the number of players, but the earned MPs were used. Thus just the medians m_i for the two halves ($i = 1, 2$) were first calculated. It turned out, however that there were quite many zeros in this sequence, as a lot of countries have not got MPs in at least 4 years out of the 7 years, that form one half. So we based our analysis on the 3rd quartiles of the halves (denoted by q_1 and q_2). As the next step these smoothed observations were normalised by the countries' actual population as well. The clustering was defined by the standardised versions of the two dimensions: q^*_1, q^*_2. Due to the outliers we had to use the ranks of these variables rather than the number of MPs.

2.2. Clustering

The chosen method was the simple k-means clustering, as in our case the data structure was rather simple and also the data size was rather small. Here the number of clusters is fixed as k, and one aims to minimize the sum of the within-cluster distances. The usual Euclidean distance was used for measuring the distance between points and cluster centres. The used method is the Hartigan–Wong algorithm [7], as implemented in the program package `R`.

It is an intricate question, how to determine the number of clusters. There is the traditional elbow-rule, which is usually of little use, as it gives no clearcut answer. The recent model-based clustering overcomes this problem, as an adapted version of the Bayesian Information Criterion (BIC, see Fraley and Raftery, [6]) may be applied successfully. The procedure is implemented in the `mclust` package of R. It is assumed in most of the cases that the observations come from multivariate normal distributions (called elliptical in this approach). Here the main question is if the covariances of the clusters are equal or different, and if different, what is the difference. Volume, shape and orientation are the three aspects considered. The model selection is based on the BIC, which is the expression

$$-2\log(L) + m\log(n),$$

where L denotes the value of the likelihood function at the optimum, n is the number of observations and m the number of parameters in the model. The model with the smallest BIC value is chosen.

3. Applications

For the chess data, the optimization procedure from above clearly prefers the 6-cluster decomposition (see Table 2), so this is the one we have applied.

place	1	2	3
model	VVV	VVV	VEV
number of clusters	6	5	8
-BIC	297.3	235.1	227.2

Table 2. The best three models for the Gaussian mixture model-based clustering for the chess data

The two models in the table:

- "VEV": ellipsoidal, equal shape
- "VVV": ellipsoidal, varying volume, shape, and orientation

The results of the clustering the countries with respect of the chess data can be seen on Figure 2. The cluster centres are shown in the respective Table 3.

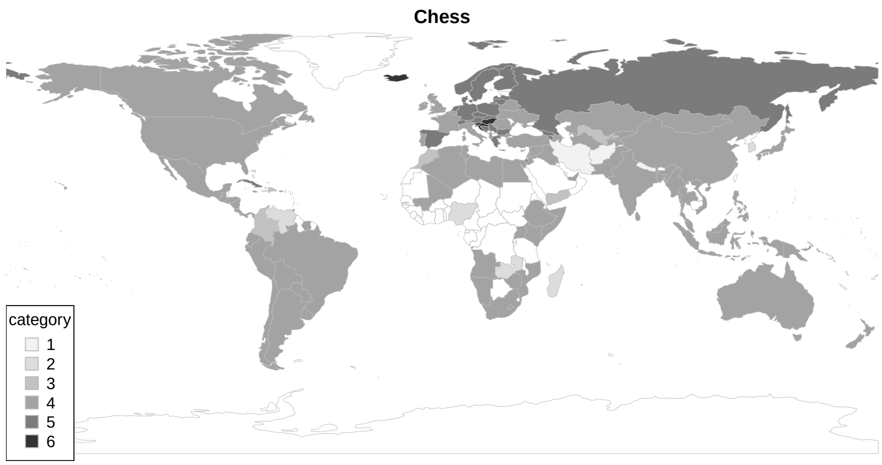

Figure 2. Map of the chess-clusters

cluster number	f_1^*	f_2^*	f_3^*	r_1	r_2	freq.
1 (very low)	0.006	0.034	0.042	6.484	1.670	3
2(low, late development)	0.017	0.021	0.051	0.977	5.794	5
3 (developing)	0.024	0.086	0.128	3.408	2.589	5
4 (medium)	0.136	0.166	0.184	1.366	2.048	77
5 (high)	0.9649	1.3073	1.4705	1.4573	2.0176	25
6 (very high)	4.0761	3.8001	3.4603	1.0152	1.5628	6

Table 3. The cluster centres for chess (all the used variables are standardized: f_i^* is the smoothed rates of players with an Elo of 2000+ in the the ith part, $r_i = f_{i+1}^*/f_i^*$)

It is worth plotting the six countries with the highest proportion of the chess players (cluster no. 6), as not all of them are visible on the map. The Figure 3 show that chess is the most popular in Iceland, by a substantial margin. Besides two small countries, Croatia, Hungary, Iceland and Slovenia form the cluster with the highest values. It is interesting, that in these countries the increase lasted only till 2008.

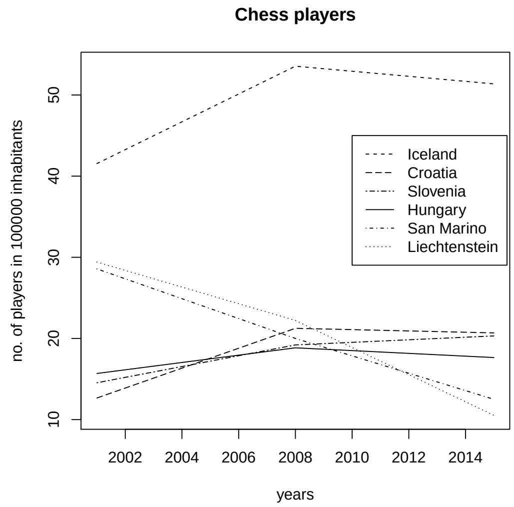

Figure 3. The smoothed rates of chess players over Elo 2000, between 2001 and 2015 in the six countries with highest rates

Our next analysis is done for the bridge data.

The best model in the Table 4: 6 clusters, "EEV": ellipsoidal, equal volume and shape and variable orientation.

place	1	2	3
model	EEV	EEV	EEV
number of clusters	6	5	7
-BIC	-1631.6	-1680.2	-1756.5

Table 4. The best three models for the Gaussian mixture model-based clustering for the bridge data

When we follow the clustering methodology from above – we have standardised the ranks, used in the bridge clustering in order to achieve them to have similar importance to the chess variables –, then we get the results of Figure 4.

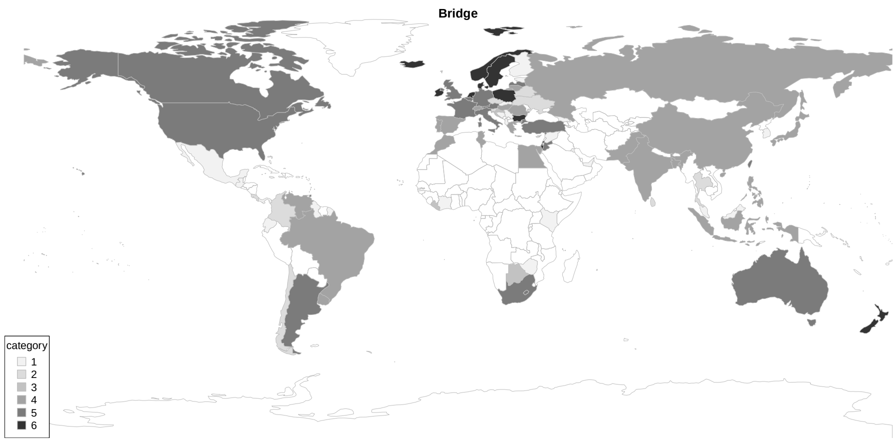

Figure 4. Map of the bridge-clusters

cluster number	q_1	q_2	freq
1 (low)	21.5	18.0	31
2 (developing)	20.0	52.1	10
3 (decreasing)	79.4	23.2	5
4 (medium)	52.6	53.0	21
5 (high)	72.6	75.0	18
6 (very high)	90.6	91.7	13

Table 5. The cluster centres for bridge (expressed as ranks)

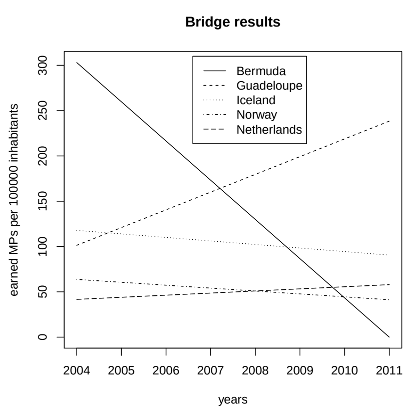

Figure 5. The smoothed number of earned bridge MPs per 100000 inhabitants in the five countries with highest rates between 2004 and 2011

For the joint dataset the model-based clustering proposed 7 clusters, but the 6-cluster model was close runner-up. We decided to use this latter one, as this provided a parallel approach to the previous ones, and besides these clusters were easy to identify. It is worth mentioning that quite a few countries were moved from the medium class in bridge to the low class here.

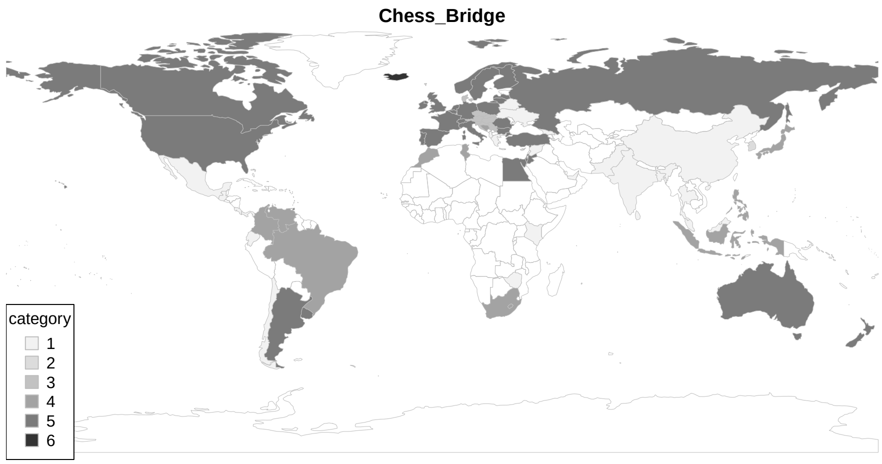

Figure 6. Map of the combined (bridge+chess) clusters

no	f_1^*	f_2^*	f_3^*	r_1	r_2	q_1	q_2
1	-0.307	-0.362	-0.377	-0.291	-0.246	-1.183	-1.183
2	-0.488	-0.569	-0.597	-1.226	7.712	-1.307	-1.307
3	1.954	1.914	1.717	-0.384	-0.512	-0.806	-0.806
4	-0.433	-0.432	-0.354	0.589	0.779	-0.612	-0.612
5	-0.025	0.054	0.102	-0.159	-0.228	-0.002	-0.002
6	6.645	7.304	7.416	-0.432	-0.396	0.575	0.575

Table 6. The cluster centres for chess and bridge (all variables are standardized, for their definitions see Section 2.1). Cluster number (no) 1 (low); 2 (chess increase: late but fast); 3 (elite chess); 4 (chess improving, bridge OK); 5 (chess OK, bridge excellent); 6 (highest level in both sports)

We can observe in Figure 6 that cluster number 2 and 6 each consist of one country: Korea and Iceland, respectively. The latter was seen already as exceptional in both sports, and Korea had by far the quickest increase in the number of chess players in the second investigated period. The other clusters show a geographical pattern: e.g. Hungary belongs together with most of its neighbours to the ”elite chess” group, where bridge is played at a medium level (see Table 3).

Next the dependence between the two clustering is investigated. When we simply check the independence of the bivariate contingency table of the clusterings, then the result does not seem to be significantly different from the independence. However, there are important classes of countries, not taken into account in this analysis: which have not been clustered in either of the two games. Completing the table with them (considered as a separate cluster, numbered as 0), the dependence becomes significant (the simulated p-value of the chi-squared test is around 0.001, Table 7.).

chess/bridge	0	1	2	3	4	5	6
0	0	8	1	4	0	1	3
1	3	0	0	0	0	0	0
2	3	1	0	0	1	0	0
3	3	0	1	0	1	0	0
4	28	14	6	0	13	14	2
5	3	6	1	0	5	3	7
6	0	2	1	1	1	0	1

Table 7. The dependence of the clustering for chess and bridge

This is mostly due to the fact that 4 out of the 5 countries forming the decreasing cluster in bridge (see Table 5) were not clustered in chess due to missing data. Another aspect, not yet taken into account is the GDP of the countries. When the number of chess players/earned bridge MPs (per capita) was analysed, a significant correlation was found between this and the countries' GDP per capita. This correlation turned out to be 0.351 for bridge and 0.509 for chess, both highly significant - as it was shown by a simple i.i.d. bootstrap procedure following Efron [5]. However, the GDP data is highly skewed, and we may suspect that the few largest observations influence this results, so we repeated the procedure, using the more robust Kendall-type rank correlation. The results are lower (0.244 and 0.460), but they are still significant ($p < 0.01$ by the bootstrap repetitions). If we repeat the same analysis for the relation between the chess and bridge results, then neither the linear, nor the Kendall-type rank correlation is significant – an interesting observation, in accordance to the findings of our cluster analysis.

4. Conclusions

Based on our analysis we can conclude that there is only weak dependence between the level of the two sports in the countries. What can be seen is the less penetration of bridge in the poorer countries as well as the exceptional performance of Iceland in both sports. However, it can also be seen that chess does not have any reserve in the highest achieving countries. Its popularity can and indeed is increasing in some Asian countries like Korea or in developing countries of Latin America. With respect to bridge as the number of MPs distributed is more or less constant over time, increase in a country can only be realised by decrease in other countries. What was to be observed is that there was no increase in the number of countries, so we cannot say that top level bridge has widened its basis.

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