LAUDATION TO
Professor Karl-Heinz Indlekofer
on his seventieth anniversary

by Imre Kátai

The time is running with us. Ten years ago we congratulated him in this journal (volume 22). During this last ten years several good things has happened with him. The most important is that he became the grandfather of two beautiful girls (Senta and Carla). Appreciating his outstanding services and research activities the senate of the Eötvös Loránd University (in 2004) honored him with the special award:

Honorary Doctor and Professor of Eötvös Loránd University.

He continued his research activity in number theory, and in probability theory. He published several interesting papers, and participated in and headed research projects, became the doctor father of young scientists: Stefan Wehmeier, Yi Wei Lee, László Germán, Anna Barát, Erdener Kaya.

We enlarge the categories to classify his new results as follows:

10. Tauberian theorems for Dirichlet- and power series
    ([120], [124], [128], [133], [134])
11. Additive arithmetical semigroups and combinatorial structures
    ([117], [127], [132])
12. Stone–Čech compactification and applications in probabilistic number theory ([92], [93], [121], [122])
13. Collaboration in DFG-projects
    ([113], [114], [115], [118], [119], [106], [125], [126], [129], [130], [131])
    - a) Law of large numbers: nontraditional approach (2002–2012)
    - b) Investigation of certain subclasses of Avakumovič, Karamata functions and their applications (2002–2013)
    - c) Rates of convergence in limit theorems of probabilistic number theory (2005-2014)
    - d.) Sums of random variables on partially ordered sets and ergodic properties of marked point processes in $\mathbb{R}^n$ (2007–2012)

10. Tauberian theorems for Dirichlet- and power series

Let $M(x,f) := \sum_{n\le x} f(n)$, $f:\mathbb{N}\to\mathbb{C}$, $f(1)=1$ be an arithmetical function.

In some of his papers he investigated $M(x,f)$ by using the following idea. Try to compare f with some function g, the mean behaviour of which is known. Let

$$M(x) := M(x, f - A_x g) = \sum_{n\le x} \left(f(n) - A_x g(n)\right), \quad A_x \in \mathbb{C}.$$

Let f be multiplicative, $g(n) = n^{ia}$ $(n\in\mathbb{N})$, $a\in\mathbb{R}$. Define λ_f and λ by the generating Dirichlet series $(s=\sigma+it, \sigma>1)$

$$F(s) = \sum f(n) n^{-s} = \exp\left(\sum_{m=2}^{\infty} \frac{\lambda_f(m)}{\log m}\cdot m^{-s}\right),$$

$$G(s) = \sum g(n) n^{-s} = \zeta(s-a) = \exp\left(\sum_{m=2}^{\infty} \frac{\lambda(m) m^{ia}}{\log m}\cdot m^{-s}\right).$$

If we assume that $|\lambda_f(m)| \le \lambda(m)$ $(=\Lambda(m)$, where Λ denotes von Mangoldt's function) and arrive at (see [120])

$$\frac{|M(x)|}{x} \le \left(\frac{1}{\log x}\int_{-\infty}^{\infty} \frac{|F(s) - A_x\,\zeta(s-ia)|^2}{|s|^2}\,dt\right)^{1/2} +$$

$$+ |A_x|\cdot \frac{1}{\log x}\sum_{m\le x} \frac{\left|\lambda_f(m) - \lambda(m)\, m^{ia}\right|}{m} + O\left(\frac{1}{\log x}\right),$$

where $s = 1 + \frac{1}{\log x} + it$.

Hence he deduces the following assertion:

Let f be multiplicative, $|f(n)|\le 1$ $(n\in\mathbb{N})$. Then the following assertions hold.

(i) Assume that the series

$$\sum_{m=2}^{\infty} \frac{\lambda(m) - \operatorname{Re}\lambda_f(m)\cdot m^{-ia}}{m\log m} \tag{10.1}$$

converges for some $a\in\mathbb{R}$. Put

$$A_x := \exp\left(\sum_{m\le x} \frac{\lambda_f(m)\cdot m^{-ia} - \lambda(m)}{m\log m}\right). \tag{10.2}$$

Then

$$\frac{1}{x}\sum_{m\le x} f(n) = A_x \cdot \frac{x^{ia}}{1+ia} + o(1)$$

as $x \to \infty$.

(ii) If the series (10.1) diverges for all $a \in \mathbb{R}$, then

$$\frac{1}{x}\sum_{n\le x} f(n) \to 0 \qquad \text{as} \quad x \to \infty.$$

This assertion is just a theorem of G. Halász, since (10.1) is convergent if and only if the same holds for

$$\sum_p \frac{1-\operatorname{Re} f(p)p^{-ia}}{p}.$$

The method he used also leads to new proofs of the prime number theorem, Wirsing's theorem and so new quantitative estimates for multiplicative functions. In [133] he uses the same idea in the case when the generating function is a power series.

11. Additive arithmetical semigroups and combinatorial structures

Earlier he investigated properties of arithmetical functions defined on (additive) arithmetical semigroups. P. Flajolet and R. Sedgewick (*Analytic Combinatorics*, Cambridge University Press, Cambridge, 2009) worked out a method for the computation of decomposable structures in combinatoric.

Indlekofer describes multisets and labeled combinatorial structures as additive arithmetical semigroups and proves limit laws for random variables arising from additive functions on these semigroups (see [134]).

12. Stone–Čech compactification of $\mathbb{N}$ and applications in probabilistic number theory

The basic idea is written in the paper [121].

It can be described as follows: $\mathbb{N}$, endowed with the discrete topology, will be embedded in a compact space $\beta\mathbb{N}$, the Stone–Čech compactification of $\mathbb{N}$, and then any algebra $\mathcal{A}$ in $\mathbb{N}$ with an arbitrary finitely additive set function (pseudomeasure) on $\mathcal{A}$ can be extended to an algebra $\mathcal{A}^*$ in $\beta\mathbb{N}$, together with an extension of the pseudomeasure on $\mathcal{A}^*$ which turns out to be a premeasure on $\mathcal{A}^*$, and to corresponding integration theory.

In a paper written by him together with A. Barát and R. Wagner ([122]) it is shown that the different compactifications of $\mathbb{Z}$ given by Novoselov in 1960 and Kubota and Sugita in 2002 are homeomorphic to a compactification introduced by Prüfer in 1925 by proving that the corresponding metrices are equivalent and generate the same topology (Theorem 1 of [122]). Choosing the algebra $\mathcal{A}$ all residues classes in $\mathbb{N}$ we arrive at another equivalent compactification of $\mathbb{Z}$ (Corollary in [122]). Further, the algebra $\mathcal{A}$ together with the asymptotic density leads via the described "integration theory" to the space of limit periodic functions of Novoselov.

13c. Rates of convergence in limit theorems of probabilistic number theory

([120], [121], [122], [124], [126], [128], [130])

This project has been headed by professors Indlekofer, Klesov and Steinebach.

They studied the deviation between the standard Gaussian distribution function Φ and a distribution function F in terms of the Lévy distance. The advantage of this approach is that their results can be applied to both probabilistic and number theoretical settings.

Let $\mathcal{L}(F,G)$ be the Lévy distance of the distribution functions F, G. O.I. Klesov and K.-H. Indlekofer proved in [130]

Theorem 1. *Let* $0 < L_0 < 1$, $p > 0$, *and*

$$\int_{-\infty}^{\infty} |x|^p \, dF(x) < \infty.$$

Denote by L *the Lévy distance between* F *and* Φ *and assume that* $0 < L \le L_0$. *Then there exists an universal constant* c_p *depending only on* L_0 *and* p *such that*

$$|F(x) - \Phi(x)| \le \frac{\lambda_p + c_p L \left(\log \frac{1}{L}\right)^{p/2}}{1 + |x|^p}$$

for all $x \in \mathbb{R}$ *where*

$$\lambda_p = \left| \int_{-\infty}^{\infty} |x|^p \, dF(x) - \int_{-\infty}^{\infty} |x|^p \, d\Phi(x) \right|.$$

Theorem 2. *Let* $p > 0$ *and*

$$\int_{-\infty}^{\infty} |x|^p \, dF(x) < \infty.$$

Denote by L the Lévy distance between F and Φ and assume that $0 < L \leq$ $\leq e^{-1/2}$. Then there exists an universal constant c_p depending only on p such that

$$|F(x) - \Phi(x)| \leq \frac{\lambda_p + c_p L \left(\log \frac{1}{L}\right)^{p/2}}{1 + |x|^p}$$

for all $x \in \mathbb{R}$ where

$$\lambda_p = \left| \int_{-\infty}^{\infty} |x|^p \, dF(x) - \int_{-\infty}^{\infty} |x|^p \, d\Phi(x) \right|.$$

Corollary. *Let a distribution function F satisfy the condition*

$$\int_{-\infty}^{\infty} x^2 \, dF(x) = 1.$$

If the Lévy distance $L = \mathcal{L}(F, \Phi)$ is such that $0 < L \leq e^{-1/2}$, then

$$|F(x) - \Phi(x)| \leq \frac{A \, L \, \log \frac{1}{L}}{1 + x^2}$$

for some universal constant $A > 0$ and all $x \in \mathbb{R}$.

They applied their results to prove

Theorem 3. *Let $\{F_n\}$ be a sequence of distribution functions such that*

$$\sup_{n \geq 1} \int_{-\infty}^{\infty} |x|^p \, dF_n(x) < \infty$$

for some $p > 1$. Let $r > 0$ and let a positive function g be such that

$$\int_{-\infty}^{\infty} \frac{g(x)}{(1 + |x|^p)^r} \, dx < \infty.$$

Then

$$\limsup_{n \to \infty} \int_{-\infty}^{\infty} g(x) |F_n(x) - \Phi(x)|^r \, dx < \infty.$$

13d. Sums of random variables on partially ordered sets and ergodic properties of marked point processes in $\mathbb{R}^n$

We worked in this project together with professors Indlekofer and Klesov.

The summary of the questions we have investigated can be classified into the following topics: strong law of large numbers for multiple sums of random variables, completely random point processes, dependent marks; renewal theorems for weighted renewal functions, random additive arithmetical functions. The results are published in [131], [136], [137], [139].