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ITERATIVE MODELLING IN H_∞ FRAMEWORK

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Abstract. We show an iterative, model based identification scheme with convergence criteria in the H_∞ framework. The robustness of the controller design respectively to the successive approximation of the plant is considered jointly with the point wise measurements and the Nevanlinna-Pick interpolation of the error transfer function.

1. Introduction

In this paper we will show an iterative, model based method with the measurement and the interpolation of the output error transfer function. The error is caused by the modelling, the error of the interpolation of the error transfer function and of course, by the measurement error. In this paper this latter is not considered, hence out method is a deterministic identification technics in the H_∞ framework. We will start from the practical requirement that one has to control and identify the plant, simultaneously. Hence, we design a model based controller for an initial model, then the output errors are measured under certain condition for the robustness of the controller which stabilizes the initial model and that also stabilizes the new, computed model. The mentioned condition is, simply, that the error transfer function obtained by interpolation over the measured data, belongs to the open unit ball of H_∞.

Hence the existence of such interpolating transfer function can be characterized by the positive semidefiniteness of the Nevanlinna-Pick matrix. If there does not exist interpolating transfer function for the measurements in the unit ball of H_∞, then we compute a new controller by the standard H_∞ optimization. Hence, the computed model and its robust controller can be considered as the initial model and controller. Then the same interative step can be repeated.

In Theorems 1,2,3 and 4 the convergence of the algorithm is proven under different hypothesis.

2. The iterative approximation

The proposed iteration is based on the simplified diagram shown at the Figure 1.

Well, at the diagram the feedback actuates for the model, while the plant is controlled by open loop which can be unfeasible for certain applications. To solve that difficulty, we can define a discrete feedback control in a general sense: updating the model and its controller iteratively by using the proposed hybrid scheme (see Figure 2).

From Figure 1 and of the closed loop of the controller and the model

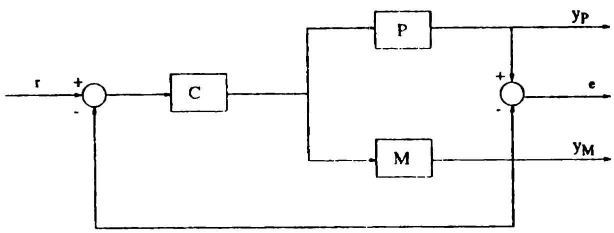

Figure 1. Diagram of the modified model based control

$$y_M = (I + MC)^{-1}MCr = MC(I + MC)^{-1} = r - (I + MC)^{-1}r.$$

Therefore, the common control u for the plant and the model is $u = C(I + MC)^{-1}r$, hence

$$y_P = PC(I + MC)^{-1}r.$$

The error of the output is

$$e = y_P - y_M = (P - M)C(I + MC)^{-1}r = Er,$$

where the error transfer function E is given by

(1) $$E = (P - M)C(I + MC)^{-1} = (P - M)(I + CM)^{-1}C.$$

If C is invertible or it has a quasi-inverse, for example, $C^*(CC^*)^{-1}$, then P can be expressed from (1) in terms of the error transfer function E

$$P = (I + E)M + EC^{-1}, \tag{2}$$

$$P = (I + E)M + EC^*(CC^*)^{-1}, \tag{3}$$

respectively. In the general case we can capture only product PC by

$$PC = (I + E)MC + E. \tag{4}$$

Now, suppose that starting from an initial model M_0 and the initial controller C_0, the error transfer function E_0 is measured and interpolated, hence, instead of using an exact transfer function $\tilde{E}_0$, we compute the plant by (2), (3) and (4) with error, respectively. Hence the computed plant this is different from the exact one. Next, we will consider that this is the subsequent model M_1. Then a new controller C_1 is given, computed, etc.

After measurement and interpolation, a new error transfer function E_1 is obtained. Hence we obtain a sequence of models $M_0, M_1, \dots$, the corresponding controllers $C_0, C_1, \dots$. The error transfer functions $E_0, E_1, \dots$, satisfy the iteration

$$M_{k+1}C_k = (I + E_k)M_kC_k + E_k. \tag{5}$$

If C_k is invertible or it has the quasi inverse $C_k^*(C_kC_k^*)^{-1}$, then

$$M_{k+1} = (I + E_k)M_k + E_kC_k^{-1}, \tag{6}$$

$$M_{k+1} = (I + E_k)M_k + E_kC_k^*(C_kC_k^*)^{-1}, \tag{7}$$

corresponding to the equations (2) and (3), respectively. If in (5) we use the same controller $C = C_0 = C_1 = \dots$, then (5) can be considered as an explicit iteration, similar to (6) and (7) with $X_k = M_kC$:

$$X_{k+1} = (I + E_k)X_k + E_k. \tag{8}$$

Hence (6), (7) and (8) has the same form. Their solutions can be computed explicitly:

$$M_k = \prod_{i=k-1}^{0} (I + E_i)(X_0 + C_0^{-1}) + \sum_{i=1}^{k-1} \prod_{j=k-1}^{i} (I + E_j)(C_i^{-1} - C_{i-1}^{-1}) - C_{k-1}^{-1}, \tag{9}$$

(10)
$$M_k = \prod_{i=k-1}^{0} (I+E_i)\left(X_0 + C_0^*(C_0C_0^*)^{-1}\right) + \sum_{i=1}^{k-1} \prod_{j=k-1}^{i} (I+E_j)\left[C_i^*(C_iC_i^*)^{-1} - C_{i-1}^*(C_{i-1}C_{i-1}^*)^{-1}\right] - C_{k-1}^*(C_{k-1}C_{k-1}^*)^{-1},$$

(11)
$$M_kC = \prod_{i=k-1}^{0} (I+E_i)(X_0C+I) - I,$$

respectively.

Lemma 1. *If* $\sum_{i=0}^{\infty} \|E_i\|_\infty < \infty$ *and* $\sum_{i=0}^{\infty} \|C_1^{-1} - C_{i-1}^{-1}\|_\infty < \infty$ *then the solution (9) of the difference equation converges at* $k \to +\infty$.

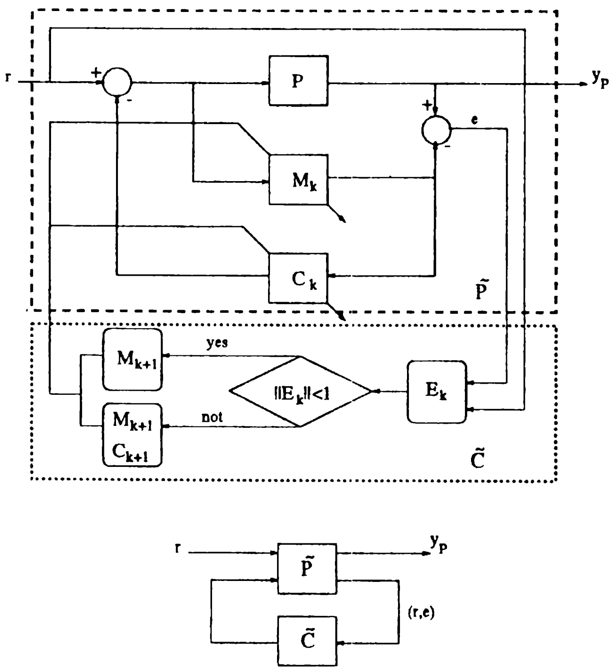

Figure 2. Hibrid closed loop controller

Proof. First we shall prove that the infinite product

$$\prod_{i=\infty}^{0}(I+E_i)=\lim_{k\to\infty}\prod_{i=k}^{0}(I+E_i) \tag{12}$$

converges in the space H_∞. However, it is a consequence of the inequality

$$\left\|\prod_{i=k}^{0}(I+E_i)-\prod_{i=l}^{0}(I+E_i)\right\|_\infty \le \prod_{i=0}^{k}(1+\|E_i\|_\infty)-\prod_{i=0}^{l}(1+\|E_i\|_\infty)$$

if $k>l$, and the well-known fact that the scalar infinite product

$$\prod_{i=0}^{\infty}(1+\|E_i\|_\infty)=\lim_{k\to\infty}\prod_{i=0}^{k}(1+\|E_i\|_\infty)$$

converges if and only if $\sum_{i=0}^{\infty}\|E_i\|_\infty<\infty$.

On the other hand, if $k>l$, then

$$\begin{aligned}
\|X_k-X_l\|_\infty &\le \|C_{k-1}^{-1}-C_{l-1}^{-1}\|_\infty+\left\|\sum_{i=l+1}^{k}\prod_{j=k-1}^{i-1}(I+E_j)(C_i^{-1}-C_{i-1}^{-1}\right\|_\infty+\\
&+\left\|\sum_{i=1}^{l}\left(\prod_{j=k-1}^{i-1}(I+E_j)-\prod_{j=l-1}^{i-1}(I+E_j)\right)(C_i^{-1}-C_{i-1}^{-1})\right\|_\infty\le\\
&\le\left(\prod_{i=0}^{k-1}(1+\|E_i\|_\infty)-\prod_{i=0}^{l-1}(1+\|E_i\|_\infty)\right)\times\\
&\times\left(\|X_0+C_0^{-1}\|_\infty+\sum_{i=1}^{\infty}\|C_i^{-1}-C_{i-1}^{-1}\|_\infty\right)+\\
&+\left(\prod_{i=1}^{k}\|C_i^{-1}-C_{i-1}^{-1}\|_\infty\right)\prod_{j=0}^{\infty}(1+\|E_j\|_\infty),
\end{aligned}$$

which tends to zero.

Remarks. 1. The solution (10) of difference equation (7) can be estimate similarly. However the corresponding conditions are

$$\sum_{i=0}^{\infty}\|E_i\|_\infty<\infty,$$

$$\sum_{i=1}^{\infty} \left\| C_i^*(C_iC_i^*)^{-1} - C_{i-1}^*(C_{i-1}C_{i-1}^*)^{-1} \right\|_{\infty} < \infty. \tag{13}$$

2. The solution (11) of the difference equation (5) converges, because the infinite product (12) is convergent if $\sum_{i=0}^{\infty} ||E_i||_\infty < \infty$.

3. Of course, the limit, in general, is not the plant P and PC, respectively. For this, we have to put further conditions.

4. If after a finite number of iteration the same controller is used, because the modeling error permits the use of the same robustly stabilizing controller, the second condition is obvious, the series is reduced to a finite sum.

Now, we consider the same iterative scheme, with further information. We have mentioned that the error transfer functions E_k are interpolated with a certain error ΔE_k. Therefore

$$M_{k+1} = P + \Delta E_k(M_k + C_k^{-1}), \tag{14}$$

$$M_{k+1} = P + \Delta E_k \left(M_k + C_k^*(C_kC_k^*)^{-1}\right), \tag{15}$$

$$M_{k+1}C = PC + \Delta E_k(M_kC + I), \tag{16}$$

respectively. From these relations the following convergence theorem can be proven easily.

Theorem 1. *Suppose that*

a) the measured error transfer functions E_k, $k = 0, 1, \ldots$, *satisfy that*

$$\sum_{k=0}^{\infty} ||E_k|| < \infty;$$

b) the controllers C_k, $k = 0, 1, \ldots$, *satisfy that*

$$\sum_{k-1}^{\infty} ||C_k^{-1} - C_{k-1}^{-1}||_\infty < \infty;$$

c) the errors ΔE_k *of the transfer functions* E_k, $k = 0, 1, \ldots$, *satisfy that*

$$||\Delta E_k(M_k + C_k^{-1})||_\infty \to 0,$$

then $\lim_{k\to\infty} M_k = P$.

The proof is the combination of the formula (14) and Lemma 1.

We notice, that if instead of (14) we suppose as the condition c) of Theorem 1, (15) and (16), then the $\lim_{k\to\infty} M_k = P$ and $\lim_{k\to\infty} M_kC = PC$ are obtained, respectively.

3. Robustness of the stabilizing controllers

Now, we consider only stabilizing controllers for the sequence of models. We are interesting to design robustly stabilizing controllers in two senses.

a) The controller which stabilizes the model, it also stabilizes the plant.

b) The same controller C stabilizes the successively computed models $M_0, M_1, \dots$. Hence the unpleasant conditions

$$\sum_{k=1}^{\infty} \|C_k^{-1} - C_{k-1}^{-1}\|_\infty < \infty,$$

$$\sum_{k=1}^{\infty} \left\|C_k^*(C_kC_k^*)^{-1} - C_{k-1}^*(C_{k-1}C_{k-1}^*)^{-1}\right\|_\infty < \infty$$

will be fulfilled automatically.

Theorem 2. *Suppose that the controller C_k stabilizes the model M_k. Then, if the estimated error transfer function E_k satisfies the inequality*

$$\|E_k\|_\infty < 1,$$

then C_k also stabilizes M_{k+1}.

Proof. We have to show that $(I + M_{k+1}C_k)^{-1}$ is stable and proper (i.e. belongs to H_∞). However, by the small gain theorem $(I + E_k)^{-1}$ exists, it is stable and proper. On the other hand

$$\begin{aligned} I + M_{k+1}C_k &= (I + E_k)M_kC + I + E_k = \\ &= (I + E_k)(I + M_kC), \end{aligned}$$

hence $(I + M_{k+1}C_k)^{-1}$ exists and

$$(I + M_{k+1}C_k)^{-1} = (I + M_kC_k)^{-1}(I + E_k)^{-1}, \tag{17}$$

therefore it belongs to H_∞.

Remarks. 1) We notice, that if $C_k(I+M_kC_k)^{-1}$ belongs to H_∞, then $C_k(I+M_{k+1}C_k)^{-1}$ also belongs to H_∞. Indeed, by (17)

$$C_k(I+M_{k+1}C_k)^{-1} = \left[C_k(I+M_kC_k)^{-1}\right](I+E_k)^{-1}.$$

2) Now, suppose that the exact error transfer function $E_k+\Delta E_k$ also satisfies the corresponding inequality

$$\|E_k+\Delta E_k\|_\infty < 1. \tag{18}$$

Then, the iterative step results the exact plant hence, by

$$(I+PC_k)^{-1} = (I+M_kC_k)^{-1}(I+E_k+\Delta E_k)^{-1},$$

it is immediate that C_k also stabilizes the plant. Analogously, if (18) holds and

$$C_k(I+M_kC_k)^{-1} \in H_\infty,$$

then also

$$C_k(I+PC_k)^{-1} \in H_\infty.$$

Now, combine the obtained condition with Theorem 1.

Theorem 3. *Consider the algorithm (6) with $C=C_0=C_1=\dots$. If C stabilizes the initial model M_0, and the interpolated error transfer functions $E_0, E_1, \dots$, satisfy that*

$$\sum_{k=0}^{\infty} \|E_k\|_\infty < \infty, \quad \|E_k\| < 1$$

and the error ΔE_k of the error transfer functions satisfy that

$$\|\Delta E_k(M_k+C^{-1})\|_\infty \to 0.$$

Then, $\lim\limits_{k\to\infty} M_k = P$, and C stabilizes the models $M_0, M_1, \dots$. If $\|E_k+$ $+\Delta E_k\|_\infty < 1$ are also satisfied, then C also stabilizes the plant.

Proof. We have to agree to the proof of Theorem 1 only the proof of the robustness. However, this can be proven by induction. The inductive step is Theorem 2. The statement that the plant is also stabilized by C, follows from Remark 2 of this section.

4. The Nevanlinna-Pick interpolation of the error transfer function

Now, we turn to the problem of the measurement and the interpolation of the error transfer function. In the practice one can measure the output errors for a given reference signal r, at certain frequencies $s_1, s_2, \ldots, s_M \in C$. For simplicity we suppose that $Res_i > 0$, $i = 1, 2, \ldots, M$. Hence we have the equations

$$E(s_i)r(s_i) = e(s_i), \quad i = 1, 2, \ldots, M. \tag{19}$$

We will suppose that the measurement is exact. Therefore $e(s_i) \in \in C^m$, $r(s_i) \in C^k$, $i = 1, 2, \ldots, M$ are the measured data, and we have to compute an interpolating transfer function $E(s)$ satisfying (19). If we consider the interpolation problem jointly with the robustness of the controller with respect to the iteration, then the condition $||E||_\infty < 1$ is the well-known Nevanlinna-Pick's interpolation, associated to the data (19).

Therefore, the existence of the interpolating error transfer function E such that the robustness condition is also satisfied, can be related to the well-known condition that the Nevanlinna-Pick matrix

$$M_{N-P} = \left(\frac{\langle r(s_i), r(s_j) \rangle - \langle e(s_i), e(s_j) \rangle}{s_i + \overline{s}_j} \right)_{i,j=1}^{k}$$

is positive semidefinite. On the other hand, if M_{N-P} is not positive semidefinite, then we have to compute an interplating error transfer function and a new stabilizing controller for the new plant, computed from the iteration. In this case, instead of the original data (19) we interpolate for the equations

$$E(s_i)r(s_i) = \frac{1}{\gamma} E(s_i)r(s_i) = \frac{1}{\gamma} e(s_i), \quad i = 1, 2, \ldots, k, \tag{20}$$

where γ is large enough, such that the corresponding Nevanlinna-Pick matrix

$$M_{N-P}(\gamma) = A - \frac{1}{\gamma^2} B = \left(\frac{\langle r(s_i), r(s_j) \rangle}{s_i + \overline{s}_j} \right) - \frac{1}{\gamma^2} \left(\frac{\langle e(s_i), e(s_j) \rangle}{s_i + \overline{s}_j} \right)$$

restricted to the image of A is positive definite. If the matrix A is positive definite, then there exists the square root $A^{1/2}$ which is also positive definite. Then it is obvious that $A - \frac{1}{\gamma^2} B$ is positive definite if and only if

$$A^{1/2} \left(A - \frac{1}{\gamma^2} B \right) A^{1/2} = I - \frac{1}{\gamma^2} A^{-1/2} B A^{-1/2}$$

is positive definite.

Therefore, if $\gamma > (\max \lambda_i)^{1/2}$, then the Nevanlinna-Pick matrix is positive definite, hence there exists a matrix-valued function $\tilde{E}(s)$ such that $||\tilde{E}||_\infty < 1$, and $\tilde{E}(s_i)r(s_i) = \frac{e(s_i)}{\gamma}$.

Therefore $E = \gamma\tilde{E}$ interpolates the original data, satisfying the inequality

$$||E||_\infty < \gamma, \qquad (21)$$

where γ^2 is greater than the maximal eigenvalue of the matrix $A^{-1/2}BA^{-1/2}$.

If A is only positive semidefinite, we need an additional property

$$ImA \supset ImB.$$

Then the same reasoning can be repeated for the matrices $\tilde{A} = A|_{ImA}, \tilde{B} = B|_{ImA} : ImA \to ImA$: $\tilde{A}$ is positive difinite, then $\tilde{A} - \frac{1}{\gamma^2}\tilde{B}$ is positive definite if and only if $I - \frac{1}{\gamma^2}\tilde{A}^{-1/2}\tilde{B}\tilde{A}^{-1/2}$ is positive definite. Therefore, if γ^2 is greater than the maximum of the eigenvalues of the matrix $\tilde{A}^{-1/2}\tilde{B}\tilde{A}^{-1/2}$, then $\tilde{A} - \frac{1}{\gamma^2}\tilde{B}$ is positive definite, that is $A - \frac{1}{\gamma^2}B$ is positive semidefinite.

Hence there exists a matrix-valued function $\hat{E}(s)$ such that $||\hat{E}||_\infty < 1$ and $\hat{E}(s_i)r(s_i) = \frac{e(s_i)}{\gamma}$. Therefore $E = \gamma\hat{E}$ interpolates the original data satisfying the inequality (21).

Remarks. 1) Which points $s_1, \ldots, s_k$ have we to interpolate $E(s)$ at? Considering further conditions, for example, the condition c) of Theorem 1, it is convenient to choose the poles $s_1, \ldots, s_p$ of the respective multipliers $M_k + C^{-1}, M_k + C^*(CC^*)^{-1}$ and $I + M_kC$. Therefore in these points $\Delta E_k(s_i) = 0$ holds automatically, hence the poles of these multipliers will be canceled by the roots of ΔE_k.

2) Other advice is to choose the reference signal r to be exciting. Then we can choose other points $s_{p+1}, \ldots, s_q$ such that the vectors $r(s_1), \ldots, r(s_q)$ form a "complete" system for the Nevanlinna-Pick interpolation.

3) If the robustness condition does not hold for E_k, M_k, C_k then C_{k+1} will be given by optimization. The stabilizing controllers for the model $M_{k+1} = D_{k+1}^{-1}N_{k+1} = \tilde{N}_{k+1}\tilde{D}_{k+1}^{-1}$ is represented as fractions by coprimes in the

module of stable and proper fractional matices. If X and Y solve the Bezout equation

$$N_{k+1}X_{k+1} + D_{k+1}Y_{k+1} = I,$$

then the Youla-Bongiorno-Jarb parametrization of the stabilizing controllers is

$$C_{k+1} = \left(X_{k+1} + \tilde{D}_{k+1}Q\right)\left(Y_{k+1} - \tilde{N}_{k+1}Q\right)^{-1},$$

where Q is arbitrary parameter in the module of the stable and proper fractional matrices of appropiate size. Then, the stability margin can be optimized by the corresponding model matching

$$\min_Q \left\|X_{k+1}D_{k+1} + \tilde{D}_{k+1}QD_{k+1}\right\|_\infty,$$

see in [4] and [6].

Let $P = D^{-1}N = \tilde{N}\tilde{D}^{-1}$ be coprime factorizations of the plant. If X, Y solve the Bezout equation $NX + DY = I$, then for the model M_0 satisfying the inequality

$$\|M_0 - P\|_\infty < \frac{1}{\min_Q \left\|XD + \tilde{D}QD\right\|_\infty} = \alpha \tag{22}$$

there exists a stabilizing controller C of P, which is also a stabilizing one for M_0.

Theorem 4. *Let M_0 an initial model for the plant P, satisfying the inequality (22). Suppose that the error ΔE_k of the measurement and the interpolation of the error transfer functions E_k satisfy the inequalities*

1) $\|\Delta E_k\|_\infty < \frac{1}{2}$,

2) $\|\Delta E_k(P + C^{-1})\|_\infty < \alpha_k < \frac{\alpha}{2}$,

then C is a robustly stabilizing controller for the plant and the recursively computed models $M_0, M_1, \dots$. If, moreover $\lim \alpha_k = 0$, *then* $\lim M_k = P$.

Proof. We will prove the statement by induction

$$\|M_0 - P\|_\infty < \alpha.$$

Suppose that the inequality

$$\|M_0 - P\|_\infty < \frac{1}{2^k}\alpha + \sum_{i=0}^{k-1} \frac{1}{2^{k-i-1}}\alpha_i$$

is proven. Then

$$\|M_{k+1}-P\|_\infty = \|\Delta E_k(C^{-1}+M_k)\|_\infty \le \|\Delta E_k(M_k-P)\|_\infty \le$$
$$\le \alpha_k + \frac{1}{2}\left(\frac{1}{2^k}\alpha + \sum_{i=0}^{k-1}\frac{\alpha_i}{2^{k-i-1}}\right) = \frac{1}{2^{k+1}}\alpha + \sum_{i=0}^{k}\frac{\alpha_i}{2^{k+1-i-1}}.$$

However

$$\frac{1}{2^k}\alpha + \sum_{i=0}^{k-1}\frac{\alpha_i}{2^{k-i-1}} \le \frac{1}{2^k}\alpha + \sum_{i=0}^{k-1}\frac{\alpha}{2^{k-i}} = \alpha,$$

hence C stabilizes the model M_k, $k=0,1,\dots$. If by our hypothesis $\alpha_k \to 0$, then

$$A_k = \frac{\alpha}{2^k} + \sum_{i=0}^{k-1}\frac{\alpha_i}{2^{k-i-1}} \to 0$$

which can be proven elementarily.

5. Examples

1. Consider the unstable plant $P(s) = \dfrac{2-s}{(1-5s)(2+s)}$ and the initial model $M_0(s) = \dfrac{1}{1-5s}$. In this case the proportional controller $C=-6$ stabilizes simultaneously the model and the plant. After 2 steps an approximating model of order 7 was obtained. The Figure 3 (a) and (b) show the Bode diagram of the models and the plant. The phase curve of the initial model is rather different from one of the plant, however the second iteration of the model is quite closed to the plant in both diagrams.

2. Now, consider the stable plant $P(s) = \dfrac{s-5}{s^2+3s+2}$ and the initial model $M_0(s) = \dfrac{s+1}{10s+1}$. It is easy to see that the proportional controller $C(s)=1$ stabilizes the model $M_0(s)$, however, it does not stabilize the plant. At the first iterative step the new model

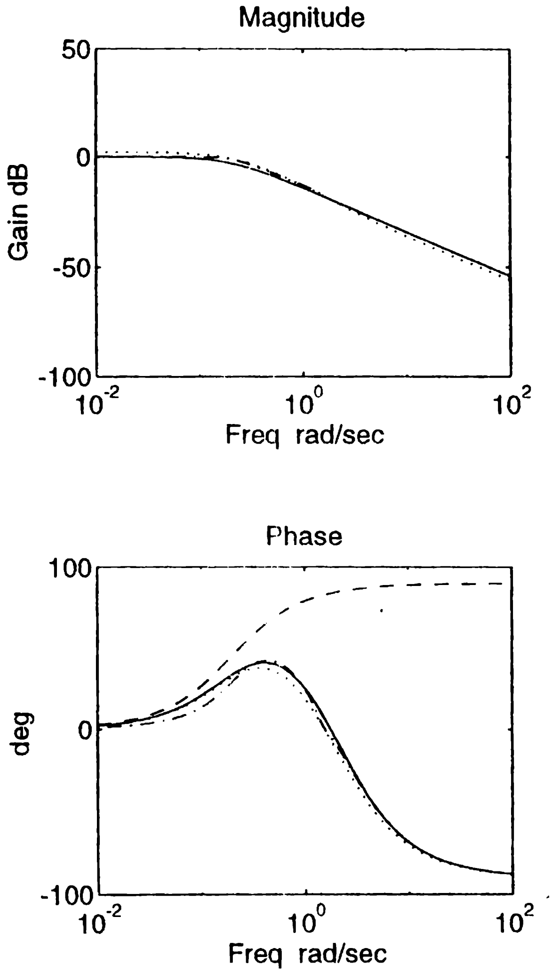

Figure 3. Bode diagrams. Plant unstable

$$M_1(s) = \frac{-0.09s^3 + 1.23s^2 - 0.25s - 12.83}{s^3 + 5s^2 + 8s + 4}$$

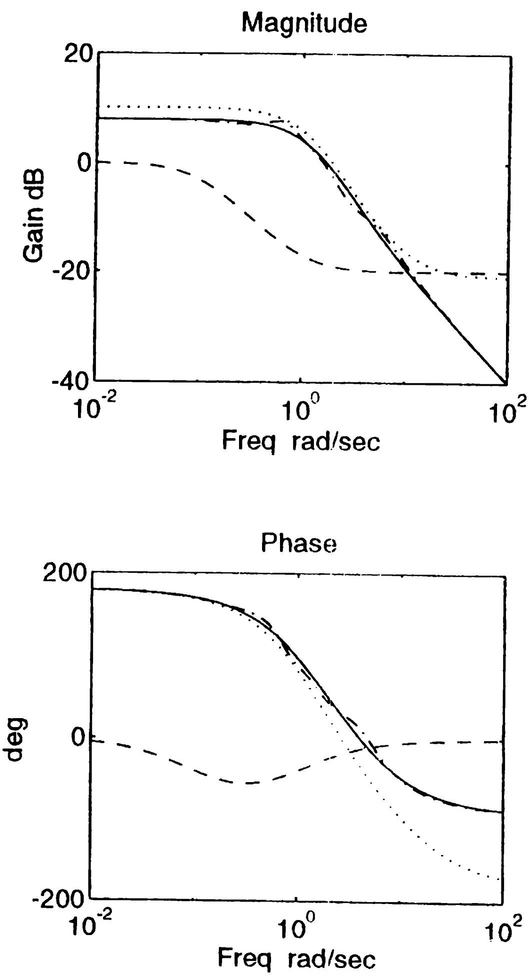

Figure 4. Bode diagrams. Plant stable

is obtained. Nevertheless neither the plant nor the new model is stabilized by the controller $C(s) = 1$. Hence we also have to compute a new controller. The controller

$$C_1(s) = \frac{s^3 + 5z^2 + 8s + 4}{1.09s^3 + 3.77s^2 + 8.25s + 16.83}$$

stabilizes both the plant and the model $M_1(s)$.

The second iteration generates a model sufficiently closed to the plant with the same robust controller $C_1(s)$, see the Bode diagrams in the Figure 4 (a) and (b).

6. Conclusion

We have given an iterative, model based algorithm which under reasonable conditions converges to the plant in H_∞. The algorithm is robust with respect to the measurement and the interpolation errors, which can be corrected at the following iteration with a correct estimate and interpolation of the error transfer function. The unique price which we have to pay that the robustness of the controller can be lost, hence we have to design a new controller. Our numerical experiences confirm the theoretical results.

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