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NUMERICAL INVESTIGATION OF THE CONVERGENCE TO THE LIMIT DISTRIBUTION IN A CYCLIC-WAITING SYSTEM

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1. Introduction

1.1. The base problem

In this paper we concentrate our attention to a question, known as "cyclic-waiting system" in the queueing theory. It is a special type of the so-called retrial systems, namely the time between two requests for service is the same fixed value in all cases.

Depending on the complexity, we have two ways to investigate the queueing systems. The first way is examination by exact mathematical methods, the second one is a model constructed by a computer program. In order to clarify our aim we describe the functioning of our system more accurately. Let us consider an airport where the entering airplanes put a landing request to the control tower upon arrival in the airside. Provided there is free system, i.e. the entering entity can be serviced at the moment of request, the airplane can start landing. However, if the server is busy, i.e. a formerly arrived plane has not accomplished landing yet or other planes are already queueing for service, then the incoming plane starts a circular maneouvre. The radius of the circle is fixed in a way that it takes the airplane T cycle time to be above the runway again, i.e. the airplane can only put further landing requests to the control tower at every nT moment after arrival, where $n \in \mathbb{N}$. Naturally, the request can only be serviced if there is no airplane queueing before it. The reception and service of the incoming planes follow the FIFO rule, according to which the earlier arriving planes are given landing permission earlier. Obviously, this system only operates properly if there are not too many planes cyclic queueing.

1.2. Our goal

Let us observe that the service process of consecutive customers does not run continuously. There are idle periods between the terminations and the beginnings of the consecutive services. It is clear that if T tends to 0 the influence of these periods will gradually decrease, thus the service process becomes continuous in the limit case. Our goal is to investigate the convergence to the limit distribution by computer simulation in order to support analytical results [11] on the basis of numerical computations.

2. Previous achievements

In [4,5] Lakatos has considered some continuous cyclic-waiting systems in which the arriving customers form a Poisson process with parameter λ, the service time distribution is either exponential with parameter μ or uniform on the interval $[c, d]$. In [6,7] similar examinations were done for discrete cyclic-waiting systems. In this case the time between two arriving requests has geometrical distribution with parameter r, the service time with parameter $1 - q$. Koba in [3] investigated the condition of the existence of ergodic distribution for a generalized system of such type. In [1,2] we have examined the abovementioned systems by computer simulation.

3. Theoretical results

Let us consider a queueing system in which the arriving requests form a Poisson process with parameter λ, the service time distribution is exponential with parameter μ (uniformly distributed on the interval $[c, d]$, where c and d are the multiples of T), and the service of a request can be started only at the moment of its arrival or at moments differing from it by the multiples of cycle time T according to the FIFO rule. Let us define an embedded Markov chain whose states correspond to the number of requests in the system at moments just before starting the service of a request $t_k - 0$ (where t_k is the moment of the beginning of the service of the $k-th$ one). The matrix of transition probabilities for this chain has the form

$$\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \cdots \\ a_0 & a_1 & a_2 & a_3 & \cdots \\ 0 & b_0 & b_1 & b_2 & \cdots \\ 0 & 0 & b_0 & b_1 & \cdots \\ \vdots & & & \vdots & \ddots \end{bmatrix},$$

whose elements are determined by the generating functions: in case of exponential service time distribution

$$A(z)=\sum_{i=0}^{\infty} a_i z^i = \frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu} z \frac{\left(1-e^{-\mu T}\right) e^{-\lambda(1-z)T}}{1-e^{-(\lambda(1-z)+\mu)T}}, \tag{1}$$

$$B(z)=\sum_{i=0}^{\infty} b_i z^i = \tag{2}$$

$$\frac{1}{(1-e^{-\lambda T})(1-e^{-[\lambda(1-z)+\mu]T})}\left\{\frac{1}{2-z}\left(1-e^{-\lambda(2-z)T}\right)\left(1-e^{-(\lambda(1-z)+\mu)T}\right)-\right.$$
$$\left.-\frac{\lambda}{\lambda(2-z)+\mu}\left(1-e^{-(\lambda(2-z)+\mu)T}\right)\left(1-e^{-(1-z)T}\right)\right\},$$

where in case of uniform service time distribution

$$\begin{aligned} A(z)=\sum_{i=0}^{\infty} a_i z^i &= \frac{e^{-\lambda c}-e^{-\lambda d}}{\lambda(d-c)}+ \\ &+\frac{z}{\lambda(d-c)}\left(e^{-\lambda c}-e^{-\lambda d}\right)\left(1-e^{-\lambda T}\right)e^{\lambda T z}\frac{1-e^{\lambda c z}}{1-e^{\lambda T z}}+ \\ &+\frac{z}{\lambda(d-c)}\lambda T e^{-\lambda(1-z)T}\frac{e^{-\lambda(1-z)c}-e^{-\lambda(1-z)d}}{1-e^{-\lambda(1-z)T}}+ \\ &+\frac{z}{\lambda(d-c)}e^{-\lambda d}\left(1-e^{\lambda T}\right)e^{-\lambda(1-z)T}\frac{e^{\lambda c z}-e^{\lambda d z}}{1-e^{\lambda T z}}, \end{aligned} \tag{3}$$

(4)
$$\begin{aligned} B(z)=\sum_{i=0}^{\infty} b_i z^i &= \frac{e^{-\lambda(1-z)c}-e^{-\lambda(1-z)d}}{[1-e^{-\lambda(1-z)T}](d-c)(1-e^{-\lambda T})}\times \\ &\times\left\{\frac{1}{\lambda(2-z)^2}\left[1-e^{-\lambda(2-z)T}+e^{-\lambda(3-2z)T}-e^{-\lambda(1-z)T}\right]+\right. \\ &\left.+\frac{T}{2-z}\left[e^{-\lambda(1-z)T}-e^{-\lambda(2-z)T}\right]\right\}. \end{aligned}$$

The generating function of ergodic distribution $P(z) = \sum_{i=0}^{\infty} p_i z^i$ for this chain has the form

$$P(z) = p_0 \frac{B(z)(\lambda z + \mu) - zA(z)(\lambda + \mu)}{\mu(B(z) - z)}, \tag{5}$$

where

$$p_0 = 1 - \frac{\lambda}{\lambda + \mu} \frac{1 - e^{-(\lambda+\mu)T}}{e^{-\lambda T}(1 - e^{-\mu T})} \tag{6}$$

in exponential case and

$$P(z) = p_0 \frac{zA(z) + (a_0 z - a_0 - z) B(z)}{a_0 (z - B(z))}, \tag{7}$$

where

$$p_0 = \frac{e^{\lambda T} - 1}{\lambda T} \left(1 - \frac{\lambda(c + d + T)}{2}\right) \tag{8}$$

in case uniform.

The condition of the existence of ergodic distribution is the fulfilment of inequality

$$\frac{\lambda}{\mu} < \frac{e^{-\lambda T}(1 - e^{-\mu T})}{1 - e^{-\lambda T}}, \tag{9}$$

and

$$\frac{\lambda(c + d + T)}{2} < 1. \tag{10}$$

As we have mentioned, intuitively it is clear that the influence of cycle time T becomes less and less as it tends to 0. Denoting the limit distribution for (5) and (7) by $P^*(z)$ in [11] are obtained the following expressions. If the service time distribution is exponential, then

$$P^*(z) = \frac{1 - \rho}{1 - \rho z},$$

i.e. the limit distribution is geometrical. In case of uniform service distribution

$$P^*(z) = \left(1 - \frac{\lambda(c+d)}{2}\right) \frac{\frac{e^{-\lambda(1-z)c} - e^{-\lambda(1-z)d}}{\lambda(d-c)}}{\frac{e^{-\lambda(1-z)c} - e^{-\lambda(1-z)d}}{\lambda(d-c)(1-z)} - z}; \tag{11}$$

it coincides with the Pollaczek-Hinchin formula in case of such distribution.

4. Computed results

The numerical investigation of convergence was realized for different values of parameters. The results are shown in Figures 1-6.

Figure 3 presents the case $\lambda = 3$, $\mu = 8$, the initial value of T is 0.2 hour, the stepsize $\Delta T = -1/300$ hour. The values of probabilities p_i $(i = 0, 1, 2, \ldots)$ are taken from the generating function (5), and the distance between $P(z)$ and the limit distribution $P^*(z)$ is measured in two different ways: the upper figure represents the values

$$|P(z) - P^*(z)| = \sum_{i=0}^{\infty} |p_i - p_i^*|,$$

the figure below

$$[P(z) - P^*(z)]^2 = \sum_{i=0}^{\infty} (p_i - p_i^*)^2$$

as the function of T. The values of first six probabilities for different steps are given in the table in Figure 1. Figure 4 shows the case $\lambda = 2$, $\mu = 6$, $T_0 = = 0.4$, $\Delta T = -1/150$.

Figures 5 and 6 represent two variants in case of uniform service time distribution, namely (1): $\lambda = 3$, $c = 0.05$, $d = 0.15$, $T_0 = 0.2$, $\Delta T = -1/300$ and (2): $\lambda = 2$, $c = 0.1$, $d = 0.2$, $T_0 = 0.2$, $\Delta T = -1/300$. The first six probability values are contained in the table in Figure 2 in the case (1).

The exact values of probabilities in the last rows of the tables were obtained in case exponential service time distribution from the corresponding Erlang formulas, in case of uniform service time distribution by using the recursive method from [9], based on the transition probabilities of embedded Markov chain.

Convergence-table for exponential distribution (Fig. 1)
$\lambda=3$, $\mu=8$, $T_0=0.2$, $\Delta T=-1/300$

	p_0	p_1	p_2	p_3	p_4	p_5
T_0	.4463389596	.1673771101	.1549594746	.9768312692e-1	.5668006684e-1	.3261449280e-1
T_1	.4506432476	.1689912180	.1545860452	.9646832002e-1	.5548072229e-1	.3165187146e-1
T_2	.4548936812	.1705851305	.1541664138	.9523865444e-1	.5429077065e-1	.3070920340e-1
T_3	.4590907670	.1721590378	.1537018274	.9399581253e-1	.5311121855e-1	.2978674433e-1
T_4	.4632350075	.1737131280	.1531935232	.9274142036e-1	.5194299777e-1	.2888468900e-1
T_5	.4673269017	.1752475880	.1526427287	.9147704882e-1	.5078696902e-1	.2800317640e-1
T_6	.4713669447	.1767626043	.1520506630	.9020421736e-1	.4964392803e-1	.2714229729e-1
T_7	.4753556287	.1782583609	.1514185339	.8892438888e-1	.4851460123e-1	.2630208991e-1
T_8	.4792934415	.1797350403	.1507475386	.8763897371e-1	.4739965297e-1	.2548254914e-1
T_9	.4831808674	.1811928251	.1500388664	.8634933208e-1	.4629968944e-1	.2468363072e-1
T_{10}	.4870183877	.1826318955	.1492936947	.8505677104e-1	.4521525723e-1	.2390525020e-1
T_{11}	.4908064799	.1840524298	.1485131887	.8376254458e-1	.4414684483e-1	.2314728530e-1
T_{12}	.4945456183	.1854546068	.1476985059	.8246786032e-1	.4309489237e-1	.2240958631e-1
T_{13}	.4982362737	.1868386027	.1468507904	.8117387348e-1	.4205978577e-1	.2169197014e-1
T_{14}	.5018789134	.1882045925	.1459711750	.7988169000e-1	.4104186222e-1	.2099422644e-1
T_{15}	.5054740025	.1895527510	.1450607829	.7859236979e-1	.4004141470e-1	.2031612214e-1
T_{16}	.5090220011	.1908832504	.1441207238	.7730692381e-1	.3905869115e-1	.1965740133e-1
T_{17}	.5125233676	.1921962631	.1431520971	.7602631728e-1	.3809389800e-1	.1901778795e-1
T_{18}	.5159785567	.1934919586	.1421559880	.7475146775e-1	.3714720025e-1	.1839698643e-1
T_{19}	.5193880204	.1947705078	.1411334726	.7348325018e-1	.3621872736e-1	.1779468728e-1
T_{20}	.5227522068	.1960320776	.1400856117	.7222249349e-1	.3530857193e-1	.1721056613e-1
T_{21}	.5260715617	.1972768356	.1390134555	.7096998417e-1	.3441679349e-1	.1664428681e-1
T_{22}	.5293465282	.1985049479	.1379180406	.6972646602e-1	.3354341943e-1	.1609550199e-1
T_{23}	.5325775454	.1997165798	.1368003920	.6849264229e-1	.3268844887e-1	.1556385749e-1
T_{24}	.5357650503	.2009118938	.1356615181	.6726917136e-1	.3185184793e-1	.1504898611e-1
T_{25}	.5389094766	.2020910535	.1345024184	.6605667704e-1	.3103356245e-1	.1455052115e-1
T_{26}	.5420112561	.2032542209	.1333240771	.6485574181e-1	.3023351098e-1	.1406808812e-1
T_{27}	.5450708163	.2044015561	.1321274644	.6366690953e-1	.2945158865e-1	.1360130875e-1
T_{28}	.5480885830	.2055332189	.1309135384	.6249068893e-1	.2868767111e-1	.1314980398e-1
T_{29}	.5510649788	.2066493672	.1296832400	.6132754855e-1	.2794161002e-1	.1271318913e-1

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	p_0	p_1	p_2	p_3	p_4	p_5
T_{30}	.5540004235	.2077501587	.1284374999	.6017792624e-1	.2721324373e-1	.1229108438e-1
T_{31}	.5568953352	.2088357506	.1271772337	.5904222438e-1	.2650239249e-1	.1188310914e-1
T_{32}	.5597501279	.2099062980	.1259033415	.5792081030e-1	.2580885899e-1	.1148888186e-1
T_{33}	.5625652138	.2109619550	.1246167097	.5681402043e-1	.2513243387e-1	.1110802548e-1
T_{34}	.5653410024	.2120028760	.1233182102	.5572215849e-1	.2447289270e-1	.1074016299e-1
T_{35}	.5680779000	.2130292124	.1220087000	.5464549884e-1	.2383000158e-1	.1038492366e-1
T_{36}	.5707763116	.2140411168	.1206890225	.5358428695e-1	.2320351623e-1	.1004194044e-1
T_{37}	.5734366382	.2150387395	.1193600046	.5253873754e-1	.2259318076e-1	.9710848554e-2
T_{38}	.5760592794	.2160222298	.1180224589	.5150903880e-1	.2199873239e-1	.9391289773e-2
T_{39}	.5786446315	.2169917369	.1166771839	.5049535336e-1	.2141990196e-1	.9082911993e-2
T_{40}	.5811930890	.2179474084	.1153249620	.4949781709e-1	.2085641331e-1	.8785368506e-2
T_{41}	.5837050436	.2188893913	.1139665598	.4851653981e-1	.2030798286e-1	.8498316068e-2
T_{42}	.5861808841	.2198178315	.1126027302	.4755160975e-1	.1977432546e-1	.8221421276e-2
T_{43}	.5886209980	.2207328743	.1112342100	.4660309137e-1	.1925515121e-1	.7954356153e-2
T_{44}	.5910257687	.2216346632	.1098617203	.4567102706e-1	.1875016733e-1	.7696799610e-2
T_{45}	.5933955794	.2225233425	.1084859674	.4475543763e-1	.1825907764e-1	.7448436071e-2
T_{46}	.5957308089	.2233990533	.1071076416	.4385632498e-1	.1778158671e-1	.7208959797e-2
T_{47}	.5980318347	.2242619379	.1057274175	.4297367010e-1	.1731739586e-1	.6978069609e-2
T_{48}	.6002990318	.2251121369	.1043459554	.4210743765e-1	.1686620845e-1	.6755474011e-2
T_{49}	.6025327720	.2259497894	.1029638983	.4125757292e-1	.1642772626e-1	.6540887057e-2
T_{50}	.6047334262	.2267750349	.1015818754	.4042400605e-1	.1600165319e-1	.6334031469e-2
T_{51}	.6069013618	.2275880106	.1002004985	.3960665012e-1	.1558769350e-1	.6134636499e-2
T_{52}	.6090369446	.2283888542	.9882036577e-1	.3880540456e-1	.1518555397e-1	.5942439453e-2
T_{53}	.6111405375	.2291777017	.9744205842e-1	.3802015371e-1	.1479494290e-1	.5757184486e-2
T_{54}	.6132125015	.2299546882	.9606614275e-1	.3725076978e-1	.1441557264e-1	.5578624783e-2
T_{55}	.6152531950	.2307199481	.9469316890e-1	.3649711078e-1	.1404715598e-1	.5406518018e-2
T_{56}	.6172629754	.2314736158	.9332367250e-1	.3575902433e-1	.1368941055e-1	.5240630804e-2
T_{57}	.6192421949	.2322158233	.9195817317e-1	.3503634676e-1	.1334205719e-1	.5080736658e-2
T_{58}	.6211912065	.2329467025	.9059717545e-1	.3432890484e-1	.1300482143e-1	.4926617299e-2
T_{59}	.6231103581	.2336663843	.8924116860e-1	.3363651547e-1	.1267743200e-1	.4778060243e-2

T=0	.6250000000	.2343750000	.8789062500e-1	.3295898438e-1	.1235961914e-1	.4634857178e-2

Figure 1

Convergence-table for uniform distribution (Fig. 2)
λ=3, c=0.05, d=0.15, T_0=0.2, ΔT=-1/300

	p_0	p_1	p_2	p_3	p_4	p_5
T_0	.5480792001	.1889832397	.1440645210	.6922228475e-1	.2911998487e-1	.1200283301e-1
T_1	.5518903531	.1902973634	.1428274439	.6762364450e-1	.2805117233e-1	.1140198569e-1
T_2	.5556478556	.1915929885	.1415609929	.6603594091e-1	.2700824282e-1	.1082476149e-1
T_3	.5593523268	.1928703278	.1402663303	.6446015923e-1	.2599121009e-1	.1027059920e-1
T_4	.5630043752	.1941295910	.1389446011	.6289724037e-1	.2500005716e-1	.9738941043e-2
T_5	.5666046043	.1953709864	.1375969307	.6134807507e-1	.2403472694e-1	.9229217912e-2
T_6	.5701536120	.1965947197	.1362244244	.5981350444e-1	.2309512519e-1	.8740853107e-2
T_7	.5736519891	.1978009957	.1348281708	.5829432772e-1	.2218112932e-1	.8273271212e-2
T_8	.5771003182	.1989900140	.1334092353	.5679129106e-1	.2129257511e-1	.7825881058e-2
T_9	.5804991789	.2001619752	.1319686676	.5530510137e-1	.2042927719e-1	.7398102265e-2
T_{10}	.5838491426	.2013170770	.1305075007	.5383642452e-1	.1959102265e-1	.6989351767e-2
T_{11}	.5871507751	.2024555123	.1290267448	.5238587864e-1	.1877756722e-1	.6599042590e-2
T_{12}	.5904046352	.2035774769	.1275273969	.5095404623e-1	.1798864843e-1	.6226597466e-2
T_{13}	.5936112755	.2046831583	.1260104293	.4954146118e-1	.1722396984e-1	.5871426586e-2
T_{14}	.5967712449	.2057727486	.1244768081	.4814863764e-1	.1648323929e-1	.5532976486e-2
T_{15}	.5998850837	.2068464315	.1229274683	.4677602951e-1	.1576611564e-1	.5210661859e-2
T_{16}	.6029533283	.2079043934	.1213633384	.4542406798e-1	.1507225825e-1	.4903926124e-2
T_{17}	.6059765080	.2089468154	.1197853241	.4409314651e-1	.1440130814e-1	.4612216935e-2
T_{18}	.6089551471	.2099738805	.1181943207	.4278362865e-1	.1375289521e-1	.4334992732e-2
T_{19}	.6118897645	.2109857666	.1165912015	.4149583812e-1	.1312663013e-1	.4071714502e-2
T_{20}	.6147808716	.2119826494	.1149768238	.4023006829e-1	.1252211354e-1	.3821854332e-2
T_{21}	.6176289757	.2129647046	.1133520298	.3898657993e-1	.1193893257e-1	.3584889515e-2
T_{22}	.6204345801	.2139321053	.1117176493	.3776561110e-1	.1137667583e-1	.3360321255e-2
T_{23}	.6231981795	.2148850225	.1100744898	.3656736006e-1	.1083490990e-1	.3147644630e-2
T_{24}	.6259202643	.2158236239	.1084233472	.3539200276e-1	.1031320166e-1	.2946373669e-2
T_{25}	.6286013217	.2167480800	.1067650061	.3423969245e-1	.9811118796e-2	.2756042911e-2
T_{26}	.6312418313	.2176585546	.1051002293	.3311054751e-1	.9328212791e-2	.2576184724e-2
T_{27}	.6338422678	.2185552111	.1034297657	.3200466259e-1	.8864034262e-2	.2406347222e-2
T_{28}	.6364031012	.2194382122	.1017543533	.3092210973e-1	.8418131035e-2	.2246089012e-2
T_{29}	.6389247959	.2203077178	.1000747119	.2986293583e-1	.7990049942e-2	.2094984750e-2

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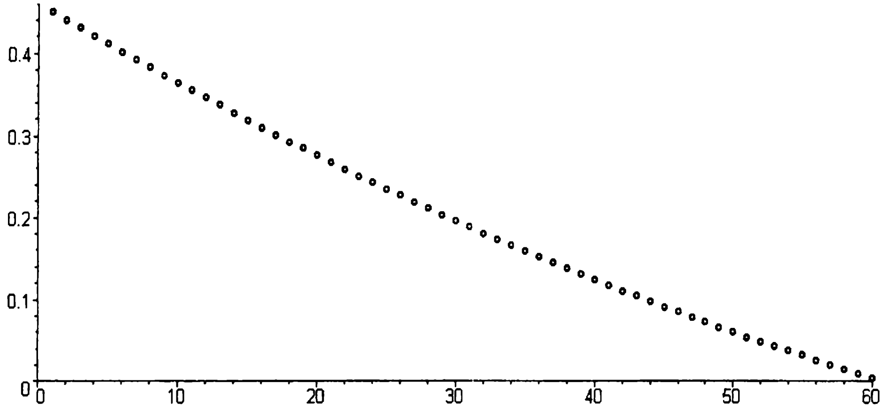
	p_0	p_1	p_2	p_3	p_4	p_5
T_{30}	.6414078137	.2211638875	.9839155217e-1	.2882716908e-1	.7579339305e-2	.1952622483e-2
T_{31}	.6438526050	.2220068763	.9670556207e-1	.2781480646e-1	.7185540006e-2	.1818598632e-2
T_{32}	.6462596233	.2228368406	.9501742391e-1	.2682583080e-1	.6808195725e-2	.1692519016e-2
T_{33}	.6486293132	.2236539336	.9332780513e-1	.2586020635e-1	.6446857934e-2	.1574014027e-2
T_{34}	.6509621133	.2244583069	.9163735291e-1	.2491786475e-1	.6101061364e-2	.1462703181e-2
T_{35}	.6532584583	.2252501098	.8994670744e-1	.2399873260e-1	.5770364149e-2	.1358246313e-2
T_{36}	.6555187784	.2260294904	.8825649245e-1	.2310270825e-1	.5454310485e-2	.1260292196e-2
T_{37}	.6577435023	.2267965971	.8656732547e-1	.2222968373e-1	.5152462283e-2	.1168518751e-2
T_{38}	.6599330481	.2275515762	.8487980341e-1	.2137952400e-1	.4864376393e-2	.1082609656e-2
T_{39}	.6620878358	.2282945671	.8319451048e-1	.2055207554e-1	.4589605338e-2	.1002243818e-2
T_{40}	.6642082738	.2290257163	.8151202632e-1	.1974718357e-1	.4327730722e-2	.9271475997e-3
T_{41}	.6662947692	.2297451614	.7983290709e-1	.1896465991e-1	.4078311095e-2	.8570207558e-3
T_{42}	.6683477303	.2304530422	.7815770631e-1	.1820431521e-1	.3840926758e-2	.7915929051e-3
T_{43}	.6703675490	.2311494963	7648696120e-1	.1746594342e-1	.3615165549e-2	.7306124863e-3
T_{44}	.6723546247	.2318346597	7482119877e-1	.1674932493e-1	.3400615333e-2	.6738258712e-3
T_{45}	.6743093421	.2325086661	.7316092699e-1	.1605421675e-1	.3196858669e-2	.6209785377e-3
T_{46}	.6762320962	.2331716514	.7150666577e-1	.1538039881e-1	.3003526454e-2	.5718732025e-3
T_{47}	.6781232566	.2338237430	.6985888618e-1	.1472758562e-1	.2820192939e-2	.5262513254e-3
T_{48}	.6799832073	.2344650731	.6821807993e-1	.1409552890e-1	.2646498152e-2	.4839325246e-3
T_{49}	.6818123212	.2350957697	.6658471491e-1	.1348394663e-1	.2482055906e-2	.4447019821e-3
T_{50}	.6836109691	.2357159616	.6495925123e-1	.1289255589e-1	.2326503763e-2	.4083785526e-3
T_{51}	.6853795064	.2363257711	.6334212673e-1	.1232104626e-1	.2179463537e-2	.3747615311e-3
T_{52}	.6871183065	.2369253271	.6173379135e-1	.1176913024e-1	.2040599795e-2	.3436961926e-3
T_{53}	.6888277234	.2375147513	.6013467183e-1	.1123649595e-1	.1909568659e-2	.3150206309e-3
T_{54}	.6905081025	.2380941629	.5854517155e-1	.1072280500e-1	.1786013627e-2	.2885585728e-3
T_{55}	.6921598035	.2386636860	.5696571140e-1	.1022774912e-1	.1669625664e-2	.2641821113e-3
T_{56}	.6937831677	.2392234386	.5539668375e-1	.9750987771e-2	.1560076383e-2	.2417422283e-3
T_{57}	.6953785163	.2397735310	.5383844884e-1	.9292144669e-2	.1457013940e-2	.2210646324e-3
T_{58}	.6969462170	.2403140890	.5229142180e-1	.8850931599e-2	.1360204766e-2	.2021033359e-3
T_{59}	.6984865843	.2408452240	.5075594485e-1	.8426954774e-2	.1269299186e-2	.1846768512e-3
T=0	0.7000000000	.2413670653	.4923243095e-1	.8019924402e-2	.1184074860e-2	.1687326822e-3

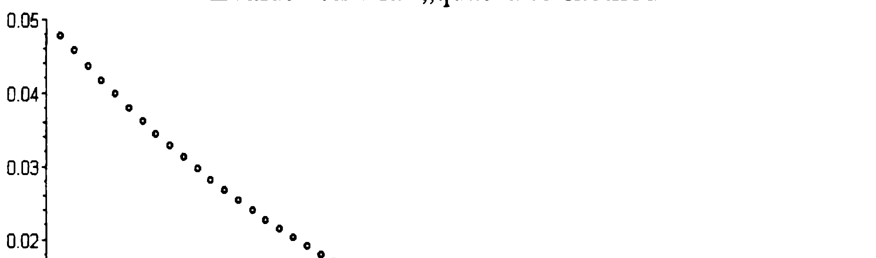
Figure 2

Evaluation with „modulus-method”



$\lambda = 3, \mu = 8, T_0 = 0.2, \Delta T = -1/300$

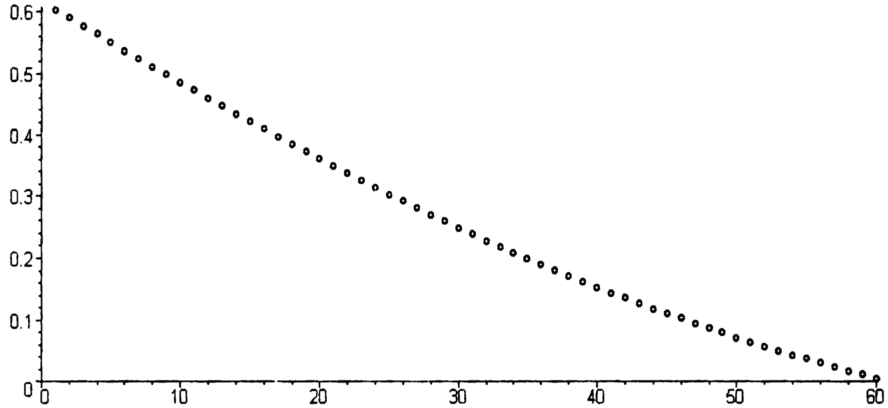
Evaluation with „quadratic-method”



$\lambda = 3, \mu = 8, T_0 = 0.2, \Delta T = -1/300$

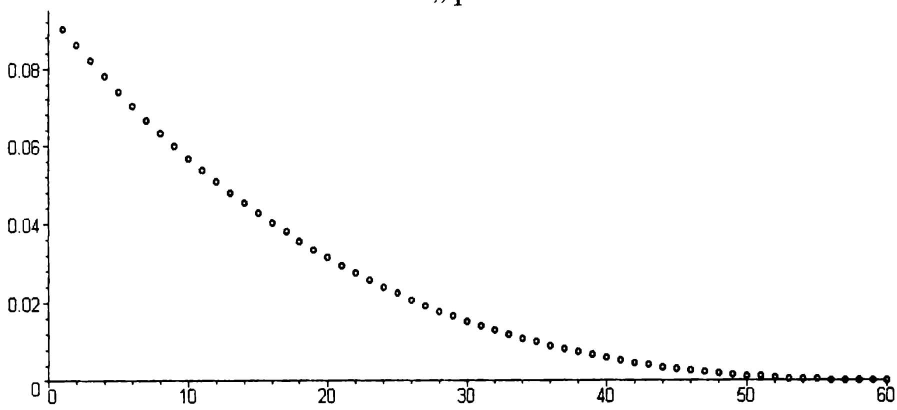
Figure 3

Evaluation with „modulus-method"



$\lambda = 2$, $\mu = 6$, $T_0 = 0.4$, $\Delta T = -1/150$

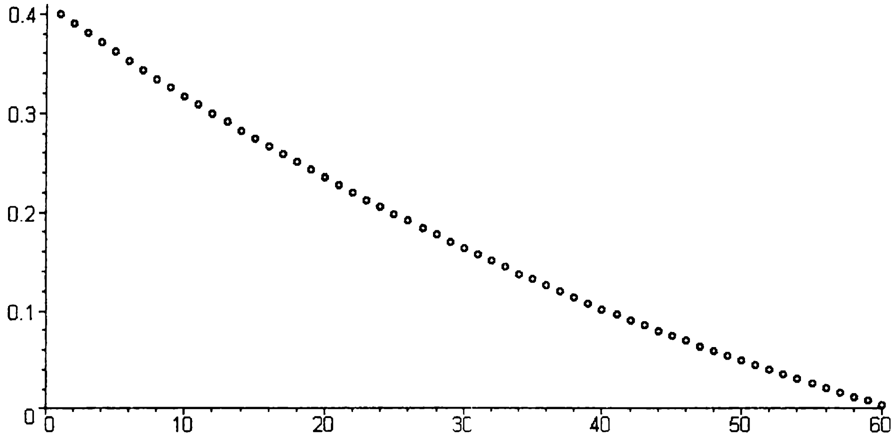
Evaluation with „quadratic-method"



$\lambda = 2$, $\mu = 6$, $T_0 = 0.4$, $\Delta T = -1/150$

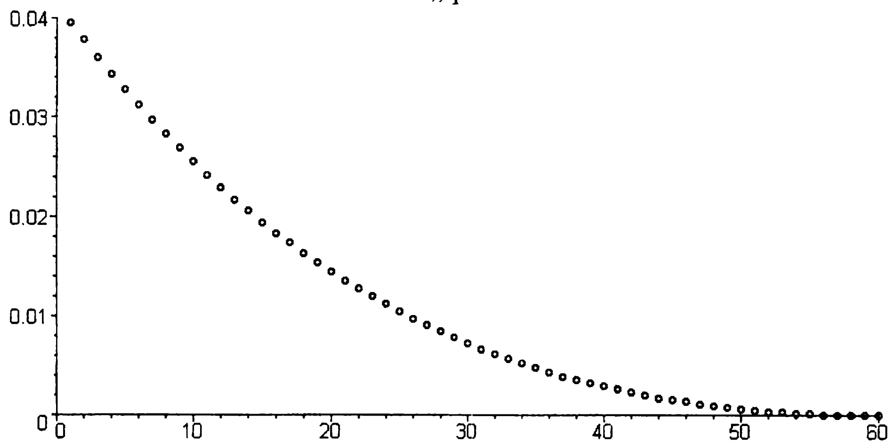
Figure 4

Evaluation with „modulus-method"



$\lambda = 3$, $c = 0.05$, $d = 0.15$, $T_0 = 0.2$, $\Delta T = -1/300$

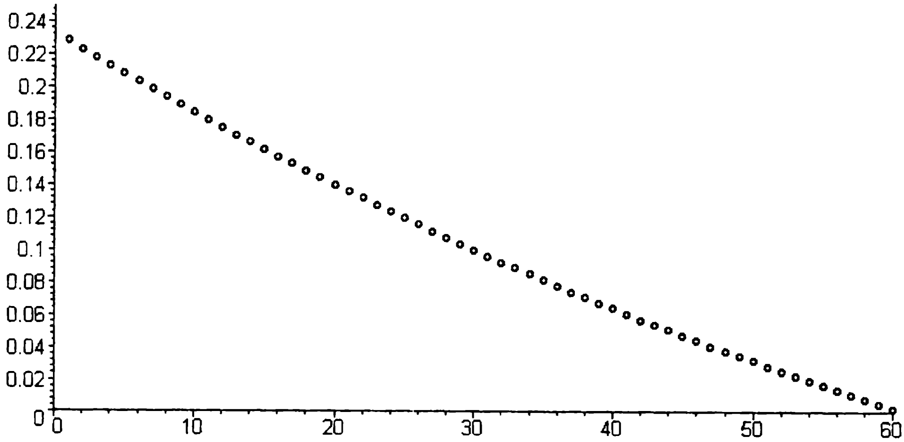
Evaluation with „quadratic-method"



$\lambda = 3$, $c = 0.05$, $d = 0.15$, $T_0 = 0.2$, $\Delta T = -1/300$

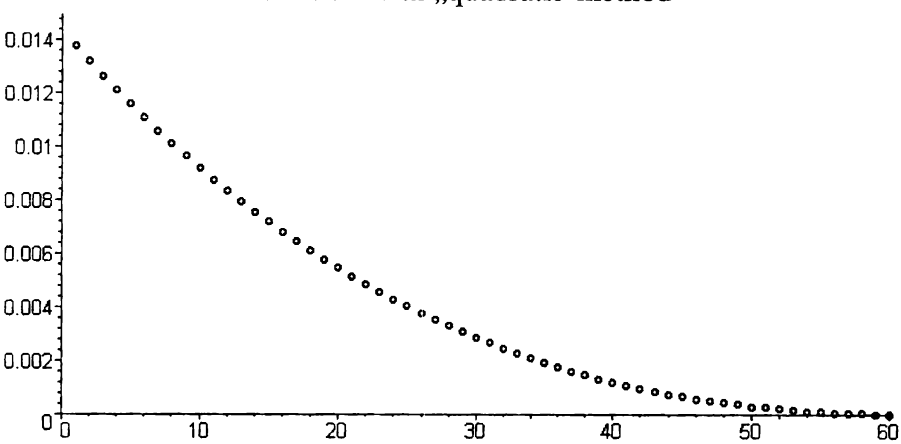
Figure 5

Evaluation with „modulus-method"



$\lambda = 2$, $c = 0.1$, $d = 0.2$, $T_0 = 0.2$, $\Delta T = -1/300$

Evaluation with „quadratic-method"



$\lambda = 2$, $c = 0.1$, $d = 0.2$, $T_0 = 0.2$, $\Delta T = -1/300$

Figure 6

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