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REMARKS ON KNOWLEDGE REPRESENTATION USING PREDICATE LOGIC

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Abstract. This paper discusses some questions of formalization of logic problems and the proving process. We deal with the following topics: identifying missing informations in problems, alternative formalizations with special regard to functions vs. predicates, predicates that can be omitted because of the non-type universe, the resolution based on natural deduction instead of "blind search", a formalization technique for finite universe consisting of named unique elements, the representation of plans with lists, solution of problems of recursive nature using resolution with equality. Finally, we present an example which is easy to solve informally but rather difficult on a formal way.

1. Introduction

In computer science we frequently face theorem proving problems. The knowledge is usually represented by the language of the predicate calculus. As a proving method the resolution or the rule based deduction can be used, among others.

The theory of theorem proving with resolution is well-known. During problem solution, however, we often encounter questions relating to representation and deduction which are beyond the scope of the theory. Solving a great number of problems interesting and sometimes surprising observations can be made.

The aim of this paper is to present some conclusions about the technique of the representation and deduction.

2. Observations about the technique of representation and deduction

Some questions of representation and deduction will be discussed below. These (and many other) problems have arisen in our educational practice.

2.1. Lack of common sence knowledge

The texts of problems often do not contain every piece of information necessary for deduction. The texts, however, are generally not considered incomplete, since we automatically add our every day knowledge.

Example 1. *Peter and Paul are the fathers of John and Tom, respectively. Both sons are sportsmen. Sportsmen are generally taller that their fathers; the only exception can be the case when the father was a basketball player. Peter used to be a basketball player and Paul did not, however, Paul is taller than Peter. Tom is shorter than John. Prove that John is taller than his father.*

The formalization of the example is the following.

(1) *Father (peter, john)*

(2) *Father (paul, tom)*

(3) *Sport (john)*

(4) *Sport (tom)*

(5) $\forall x \forall y[Father(x,y) \land Sport(y) \land \neg Basketball(x) \rightarrow Taller(y,x)]$

(6) *Basketball (peter)*

(7) ¬Basketball(paul)

(8) *Taller (paul, peter)*

(9) *Shorter (tom, john)*

(G) *Taller (john, peter)*

The goal statement (G) cannot be proved using only the axioms (1)-(9). The text of the example lacks statements. The connection between the relations *Shorter* and *Taller* should be given. In this case one side of the connection is sufficient.

(10) $\forall x \forall y[Shorter(x,y) \rightarrow Taller(y,x)]$

The other missing information is the transitivity of the relation *Taller*.

(11) $\forall x \forall y \forall z[Taller(x,y) \land Taller(y,z) \rightarrow Taller(x,z)]$

From (1)-(11) the goal formula (G) already logically follows. The proving process will be discussed in 2.4. We note that the statement (6) is unnecessary in the proving process.

More extreme examples for missing information are given in 2.5, 2.6, and 2.7.

2.2. Alternative formalizations of problems

A given problem often can be formalized in more than one way. A typical choice in the formalization is whether an attribute of an object is to be represented by a predicate or by a function. A simple example is the next one.

Example 2. *Daughters are prettier than their mothers. Show that Mary is prettier than her mother.*

In the first case *mother(x)* denotes the mother of person x. The formalization is the following.

(1) $\forall x Prettier(x,\ mother(x))$

(G) $Prettier\ (mary,\ mother\ (mary))$

(Negating (G) the resolution process will terminate in one step.)

In the second case *Mother (x, y)* denotes that x and y are related as mother and daughter. The formalization in this case goes as follows.

(1) $\forall x \forall y [Mother(x,y) \rightarrow Prettier(y,x)]$

(G) $\forall x [Mother(x, mary) \rightarrow Prettier(mary, x)]$

The second version, as usual, is a bit more complicated than the first one. The reason is the difference between the two approaches. In the first case a "natural Skolemization" was carried out by the introduction of the function *mother (x)*. In the second case the Skolemization is hidden in the formulas. The negation of the goal statement (G)

(G') $\exists x [Mother(x, mary) \wedge \neg Prettier(mary, x)]$

exactly represents that Mary has a mother (but the daughter is not prettier than her mother).

2.3. Predicates that can be omitted

The text of the problem usually suggests which predicates should be introduced. A well known example is the following [1].

Example 3. *Some patients like all doctors. No patient likes any quack. Show that no doctor is a quack.*

The next formalization [1] closely follows the text.

(1) $\exists x \{P(x) \wedge \forall y [D(y) \rightarrow L(x,y)]\}$

(2) $\forall x \forall y \{[P(x) \wedge Q(y)] \rightarrow \neg L(x,y)\}$

(G) $\forall y[D(y) \rightarrow \neg Q(y)]$

A second version can be obtained if we do not use the predicate $P(x)$. It can be omitted because the first argument of the predicate $L(x, y)$ always denotes a patient.

(1) $\exists x \forall y[D(y) \rightarrow L(x, y)]$
(2) $\forall x \forall y[Q(y) \rightarrow \neg L(x, y)]$
(G) $\forall y[D(y) \rightarrow \neg Q(y)]$

It can be easily seen that this simpler formalization is also adequate to the problem. We note that predicates $D(y)$ and $Q(y)$ cannot be dropped, since the second argument of $L(x, y)$ is of an alternative type: it denotes either a doctor or a quack.

It is generally true that the introduction of a predicate for the whole domain of discourse is unnecessary. Furthermore it is also needless to apply a predicate for the description of a data type set if its elements are not "mixed" with that of an other. (The redundant formalization, as in the above example, can be easier to understand.)

2.4. Comparision of resolution and natural deduction

The resolution process usually is not as easy to understand as the natural deduction. The reason is on one hand that it works with clauses and on the other hand that the resolution is a refutation method. Therefore, one could get an impression that the resolution can only be carried out in an automatic way by a "blind search". However, this is often not the case.

The steps of resolution frequently have a clear meaning, more over it is definitely advisable to base the resolution on the natural deduction.

Reconsidering Example 1 we see that several matches of the transitivity law (11) are possible. Therefore, it is the best to begin with an informal proof. The main steps of it are the following.

1. It is a fact that John is taller than Tom.

2. Tom is taller than Paul according to the rules (2),(4),(7), and (5).

3. John is taller than Paul according to the transitivity law.

4. It is a fact that Paul is taller than Peter.

5. Applying the transitivity law again we conclude that John is taller than Peter.

After this proving process it is easy to find the appropriate steps of the resolution. The main steps are than the following.

(12) Taller (tom, paul) resolvent of (5),(2),(4),(7)

(13) Taller (john, paul) resolvent of (9),(10),(11),(12)
(14) Taller (john, peter) resolvent of (11),(8),(13)
(15) NIL resolvent of (14),(G')

It is generally advisable to solve logical problems at first on a natural way, and than to use it as a guide line for resolution.

2.5. Problems with finite and defined universe

Consider the following example.

Example 4. *Cubes A, B, and C are arranged on a table. Cubes A and B are lying on the table. The top of cube B is clear. Cube C is not on the table. Prove that cube C is on cube A.*

In this problem the domain is finite and consists of defined elements. In such cases there are two possible ways of formalization. The first is that quantified rules are given without quantifiers specialized for every element of the universe. As an example consider a rule and its formalization.

If a cube is on the table, it cannot be on the top of an other cube.

$[Ontable(a) \rightarrow \neg On(a,b) \wedge \neg On(a,c)] \wedge$
$[Ontable(b) \rightarrow \neg On(b,a) \wedge \neg On(b,c)] \wedge$
$[Ontable(c) \rightarrow \neg On(c,a) \wedge \neg On(c,b)]$

This approach is completely ineffective. The right solution is to retain quantifiers in the rules. In this case, however, the elements of the domain are to be listed in a statement. Accordingly, the rules are to be transformed introducing the equality sign. The formalization of the above example completed with the necessary rules is the following.

(1) $\forall x[Ontable(x) \vee \exists y On(x,y)]$
(2) $\forall x \forall y \forall z[Ontable(x) \wedge On(y,z) \rightarrow \neg x = y]$
(3) $\forall x[Clear(x) \vee \exists y On(y,x)]$
(4) $\forall x \forall y \forall z[Clear(x) \wedge On(y,z) \rightarrow \neg x = z]$
(5) $\forall x \forall y[On(x,y) \rightarrow \neg x = y]$
(6) $\forall x[x = a \vee x = b \vee x = c]$
(7) Ontable (a)
(8) Ontable (b)
(9) Clear (b)
(10) $\neg$ *Ontable(c)*
(G) $\exists x[On(c,x) \wedge x = a]$

The proof can be found upon the suggestion given in 2.4, following the steps of natural deduction. It is to be noted that statements (2), (3), (7), and (8) are not used in the resolution process.

2.6. A complex example: the towers of Hanoi

In this part the well-know problem of the towers of Hanoi is discussed.

Example 5. *Towers of Hanoi with two disks, for the simplicity. (The problem is supposed to be known.)*

A possible solution of the problem is the following [4]. The replacement of the upper disk from peg i to peg j is denoted by $m(i,j)$. The solution will be a list of such moves. Predicate $A(x,y,z)$ expresses that the list z is the concatenation of the lists x and y. Predicate $H(n,i,j,k,x)$ holds if the transposition of n number of disks from peg i to peg j (using peg k) is accomplished with list x. The formalized version of the example, omitting the universal quantifiers, is the following:

(1) $H(1,i,j,k,m(i,j).nil)$

(2) $H(m,i,k,j,y) \wedge H(1,i,j,k,m(i,j).nil) \wedge H(m,k,j,i,z) \wedge$
$A(y,m(i,j).z,x) \wedge m = n-1 \rightarrow H(n,i,j,k,x)$

(3) $A(nil,r,r)$

(4) $A(u,v,w) \rightarrow A(s.u,v,s.w)$

(5) $x = x$

(G) $\exists x H(2,1,2,3,x)$

In this example three interesting questions arise.

The towers of Hanoi is a special planning problem. When solving planning problems, the usual way is to introduce a variable for the states, and this variable is one argument of the functions and predicates. In this case the solution is obtained as a composition of functions [1,2,3]. In our representation the solution is produced in the form of a list. Accordingly, the axioms of list (3) and (4) should be given.

In Example 5 the equality sign has already been used in order to identify the cube that is actually considered. The role of the equality sign in our problem is somewhat different. The evaluation of predicate $H(n,...)$ (for $n > 1$) is reduced to the evaluation of $H(n-1,...)$ in a recursive way. A well-known problem of this type is to compute the value of n! according to the recursive rule $Fact(n) = n * Fact(n-1)$ [3]. The logical substitution in itself cannot support this way of computing. This is why rule (1) is stated so that the composed term $n-1$ is placed only at one side of the equality sign. For this reason the axiom of equality (5) is given, too. There are also further approaches to formalizations of this type [3].

The third question is also related to the weakness of the original resolution method. The value of the term $2-1$ cannot be computed in the frame of the pure resolution. For this purpose procedural attachment is to be used.

The solution can be obtained through a usual question-answering procedure. The negated goal statement is completed to tautology, and then it will be resolved with the clause form of the following formulas:

$$(2),\ (5),\ \mathit{proc.\ att.},\ (1),(1),(1),(4),(3)$$

The solution is formed by substitutions in x of H:

$$m(1,3).\ m(1,2).\ m(3,2).nil$$

2.7. Problems difficult to represent

There are certain problems which can easily be solved by human intuition but can be formalized only with great efforts. Finally, we present an example of this type. This was one of the most difficult problems to formalize for my students.

Example 6. *Consider an arrangement of cubes and pyramids on the table. We know that on the top of each cube there is an other cube or a pyramid. Show that there is a pyramid above each cube.*

This problem is indeed quite simple for human thinking. The axioms and the goal statement of this block world problem are the following.

(1) $\forall x[Cube(x) \vee Pyramid(x)]$
(2) $\neg\exists x[Cube(x) \wedge Pyramid(x)]$
(3) $\forall x[Cube(x) \rightarrow \exists y On(y,x)]$
(4) $\forall x[Pyramid(x) \rightarrow \neg\exists y On(y,x)]$
(5) $\forall x\forall y\{Above(y,x) \rightarrow [On(y,x) \vee \exists z On(z,x) \wedge Above(y,z)]\}$
(6) $\forall x\{Cube(x) \rightarrow \exists y[Above(y,x) \wedge \neg\exists z On(z,y)]\}$
(G) $\forall x\{Cube(x) \rightarrow \exists y[Pyramid(y) \wedge Above(y,x)]\}$

Formula (5) defines the connection between the predicates *On* and *Above*. Rule (6) expresses that a structure of blocks has a finite size. The empty clause can be derived from the above axioms and the negated goal statement.

3. Conclusion

The present paper is the achievement of the educational practice. We have been solving logical problems during a semester with students. During this tutorial several interesting ideas turned up and some of them are outlined in this article.

Three of these topics: resolution and equality, representation problems with lists, and different representations of planning problems deserve further investigation in the neer future.

Hopefully, the above explained methods will be of practical use for the reader and contribute to deeper understanding of logic.

References

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