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SOUND SYSTEM OF *LPC+Ch*, THE LOGIC OF MODIFIERS

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Abstract: The paper discusses a *predicate logic* of *modifier operators*. A formal semantics of modifier logic is given, and a sound axiom system for that is developed. We call this logic *a system of LPC+Ch* (standing for the words 'Lower Predicate Calculus with additional Characteristics of modifiers'). Some alternatives for the application topics, and both similarities and differences with some non-classical logics are discussed.

Keywords: Modifiers, predicate calculus, logic of hedges, modal logic, model theory, axiomatic system

1. *Ch*-STRUCTURES

The *alphabet* of the *Ch-language* are as follows. The connectives $\neg$ standing for negation, $\rightarrow$ standing for implication, as primitives, and connectives $\wedge$ standing for disjunction, $\vee$ standing for conjuction, and $\leftrightarrow$ standing for equivalence are derived from those in the known classical way. We adopt the set of predicate symbols $\mathbb{P} = \{p_i^n \mid n = 1, 2, \ldots;\ i = 0, 1, 2, \ldots\}$ and the set of function symbols $\mathbb{F} = \{f_i^n \mid n = 1, 2, \ldots;\ i = 1, 2, \ldots\}$ straight from *LPC*. We further need some added characters for formalizing a set of *characteristic operator symbols* $\mathcal{O} = \{\mathfrak{I}, \mathcal{F}_1, \mathcal{F}_2, \ldots\}$ where the operators $\mathcal{F}_1, \mathcal{F}_2, \ldots$ are *substantiating* and $\mathfrak{I}$ is an *identity operator*. We can denote these *modifier operators* by metavariables $\mathcal{H}, \mathcal{F}, \mathcal{V}, \ldots$ (with or without numerical subscripts). For any modifier $\mathcal{F} \in \mathcal{O}$, we can form its *dual modifier* $\mathcal{F}^* = \neg\mathcal{F}\neg$, and the set of duals we symbolize by $\mathcal{O}^*$. Clearly $\mathcal{F}^{**} = \mathcal{F}$. For the indentity operator $\mathfrak{I}$ it holds $\mathfrak{I}^* = \mathfrak{I}$. Modifiers belonging to $\mathcal{O}^*$ are called *weakening operators*. The formation of *well-formed formulae (wffs)* is as follows:

Definition 1.1. If $\mathcal{S} = (\mathbb{F}, \mathbb{P}, \mathcal{O})$ is the symbolic alphabet, then the set of $\mathcal{S}$-formulae, or wffs, $\mathbf{W}_{\mathcal{S}}$ is the smallest set $\mathbf{W}$ for which it holds

(1°) the set W of wffs of *LPC* is a subset of $\mathbf{W}$;

(2°) if $\alpha \in \mathbb{W}$ and $\mathcal{F} \in \mathbb{O}$ then $\mathcal{F}(\alpha) \in \mathbb{W}$;
(3°) if $\alpha \in \mathbb{W}$ then $\neg\alpha \in \mathbb{W}$;
(4°) if $\alpha, \beta \in \mathbb{W}$ then $(\alpha \rightarrow \beta) \in \mathbb{W}$;
(5°) if $\alpha \in \mathbb{W}$ and x is a variable then $\forall x : \alpha \in \mathbb{W}$.
(6°) All the wffs are generated by the steps (1°) – (5°). □

We give the *model theory* of the *Ch*-language as follows:

Definition 1.2. A (*Kripke-type*) *Ch-structure* is an ordered triple

$$\mathcal{U} = \langle \mathcal{P}(U), \mathbf{R}, \varphi \rangle,$$

where U is a *universe of worlds*, $\mathcal{P}(U)$ is the power set of U, $\mathbf{R}$ is any relation on U, so-called *accessibility relation* for U, and φ is a function $\varphi : \mathbb{N}_0 \rightarrow \mathcal{P}(U)$ which defines for each n-ary predicate symbol p_i^n, $i \in \mathbb{N}_0$ a set $\varphi(i)$ of world where p_i^n is true for a given assignment of elements $a_1, \ldots, a_n$ to $x_i, \ldots, x_n$. □

So φ defines for each world $u \in U$ a set of predicates, which are true in u for a given assignment, i.e. a set

$$\{p_i^n \mid u \in \varphi(i)\}.$$

The corresponding set for a combined wff α is

$$\{\alpha \mid u \in \varphi_\alpha(i_1, \ldots, i_k),\ i_j \in \mathbb{N}_0,\ j = 1, \ldots, k\},$$

where α consists of k atoms. For any $\mathbf{R}$ there is also a subrelation system

$$\mathbf{R}_1 \subset \mathbf{R}_2 \subset \ldots \subset \mathbf{R}_n \subset \ldots \subset \mathbf{R}$$

associated with the modifiers $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_n, \ldots$ Further, $\psi(u)$ is the *domain* of u. Intuitively it means that $\psi(u)$ is the set of all individual constants existing in u. A world system corresponding to $\mathcal{U}$ is an ordered pair

$$\mathcal{K} = \langle \mathcal{P}(U), \mathbf{R} \rangle$$

Definition 1.3. For each wff $\alpha \in \mathbb{W}$ and each modifier $\mathcal{F} \in \mathbb{O}$ there is a mapping, the *universe of $\mathcal{F} \in \mathbb{O}$ corresponding to $\alpha \in \mathbb{W}$*,

$$\eta : \mathbb{W} \times \mathbb{O} \rightarrow \mathcal{P}(U)$$

such that

(i) $\eta(p_i^n, \mathfrak{I}) = \varphi(i)$;

(ii) if $\eta(\alpha, \Im) = A$ then for each $\mathcal{F}_j \in \mathcal{O}$ there is
$\eta(\alpha, \mathcal{F}_j) = A \cup \{u \mid \text{for all } t \in \varphi_\alpha(i_1, \ldots, i_k),\ t\mathbf{R}_j u\}$;

(iii) if $\eta(\alpha, \mathcal{F}_j) = A$ then $\eta(\neg\alpha, \mathcal{F}_j) = \overline{A}$;

(iv) $\eta(\alpha, \mathcal{F}_j) = A$, and $\eta(\beta, \mathcal{F}_j) = B$ then $\eta(\alpha \to \beta, \mathcal{F}_j) = \overline{A} \cup B$;

(v) $\eta(\forall x \quad \alpha, \mathcal{F}_j) = A$, where for all $u \in A$, $x \in \psi(u)$. □

It is natural to define the mapping $\eta : \mathbb{W} \times \mathcal{O}^* \to \mathcal{P}(U)$ in similar way such that $\mathcal{F}_j$ and the corresponding $\mathcal{F}_j^*$ has the same universe.

Definition 1.4. Given two modifiers $\mathcal{F}_j, \mathcal{F}_k \in \mathcal{O}$, if for any $\alpha \in \mathbb{W}$, $\eta(\alpha, \mathcal{F}_j) \subseteq \eta(\alpha, \mathcal{F}_k)$ in a *Ch*-structure $\mathcal{U}$, we say that $\mathcal{F}_k$ is *at least as strong as* $\mathcal{F}_j$ in $\mathcal{U}$, abbreviated by $\mathcal{F}_j \precsim^{\mathcal{U}} \mathcal{F}_k$.

For the corrsponding duals $\mathcal{F}_j^*, \mathcal{F}_k^* \in \mathcal{O}^*$, if for any $\alpha \in \mathbb{W}$, $\eta(\alpha, \mathcal{F}_k^*) \subseteq \eta(\alpha, \mathcal{F}_j^*)$ in a *Ch*-structure $\mathcal{U}$ it holds $\mathcal{F}_j^* \precsim^{\mathcal{U}} \mathcal{F}_k^*$.

If $\mathcal{F}_j \precsim^{\mathcal{U}} \mathcal{F}_k$ holds for all structures $\mathcal{U}$, we say that $\mathcal{F}_k$ is *at least as strong* as $\mathcal{F}_j$, abbreviated by $\mathcal{F}_j \precsim \mathcal{F}_k$, where either $\mathcal{F}_j, \mathcal{F}_k \in \mathcal{O}$ or $\mathcal{F}_j, \mathcal{F}_k, \in \mathcal{O}^*$. □

We give the following truth definitions:

Definition 1.5. The *truth* of a wff α *in a world* $u \in U$ of $\mathcal{U}$ abbreviated by $\models_u^{\mathcal{U}} \alpha$, is defined recursively as follows:

(i) $\alpha \equiv p_i^0 \Rightarrow \models_u^{\mathcal{U}} \alpha$ iff $u \in \varphi(i)$;

(ii) $\alpha \equiv p_i^n(x_1, \ldots, x_n) \Rightarrow \models_u^{\mathcal{U}} \alpha$ iff, given an assignment of elements $a_1, \ldots, a_n \in S^n$ to $x_1, \ldots, x_n$, where $S \subset \bigcup_{u \in U} \psi(u)$, the n-tuple $(a_1, \ldots a_n) \in \varphi(i)$.

(iii) $\alpha \equiv \neg\beta \Rightarrow \models_u^{\mathcal{U}} \alpha$ iff not $\models_u^{\mathcal{U}} \beta$;

(iv) $\alpha \equiv \beta \to \gamma \Rightarrow \models_u^{\mathcal{U}}$ iff $\models_u^{\mathcal{U}} \beta \Rightarrow \models_u^{\mathcal{U}} \gamma$,

(v) $\alpha \equiv \Im(\beta) \Rightarrow \models_u^{\mathcal{U}} \alpha$ iff $\models_u^{\mathcal{U}} \beta$;

(vi) $\alpha \equiv \mathcal{F}(\beta) \Rightarrow \models \mathcal{U}_u \alpha$ iff $A \in \mathcal{P}(U)$ is a universe of $\mathcal{F}$ corresponding to α, $u \in A$, and for all $t \in A$ such that $u\mathbf{R}t$ it follows $\models_t^{\mathcal{U}} \beta$.

(vii) $\alpha \equiv \forall x \quad \beta(x) \Rightarrow \models_u^{\mathcal{U}} \alpha$ iff $\models_u^{\mathcal{U}} \beta(a)$ for every assignment of a , where $a \in \psi(u)$. □

By Definition 1.3.(v) it can be derived the corresponding truth rule for the dual $\mathcal{F}^* = \neg\mathcal{F}\neg$ of the operator $\mathcal{F}$ as follows:

(viii) $\models_u^{\mathcal{U}} \mathcal{F}^*(\alpha)$ iff $A \in \mathcal{P}(U)$ is a universe corresponding to $\mathcal{F}^*$, $u \in A$, and there exists $t \in A$ such that $u\mathbf{R}t$ and $\models_t^{\mathcal{U}} \alpha$. □

The notions *validity* and *consistency* are first defined in a world system and then generally.

Definition 1.6. (1°) α is *valid in a world system* $\mathcal{K} = \langle \mathcal{P}(U), \mathbf{R} \rangle$, abbreviated by $\models^{\mathcal{K}} \alpha$ iff $\models_u^{\mathcal{U}} \alpha$ for all *Ch*-structures $\mathcal{U}$ of $\mathcal{K}$ and $u \in U$.

(2°) α is *consistent in a world system* $\mathcal{K} = \langle \mathcal{P}(U), \mathbf{R} \rangle$, iff $\models_u^{\mathcal{U}} \alpha$ for some *Ch*-structures $\mathcal{U}$ of $\mathcal{K}$ and $u \in U$.

(3°) α is *valid*, denoted by $\models \alpha$ iff α is valid in all world systems $\mathcal{K} = \langle \mathcal{P}(U), \mathbf{R} \rangle$.

(4°) α is *consistent*, iff a is consistent in some world system $\mathcal{K}$. □

(3°) and (4°) can be equivalently defined as follows:

(a) $\models \alpha$, iff $\models_u^{\mathcal{U}} \alpha$ for all $\mathcal{U} = \langle \mathcal{P}(U), \mathbf{R}, \varphi \rangle$, $u \in U$.

(b) α is *consistent*, iff $\models_u^{\mathcal{U}} \alpha$ for some $\mathcal{U} = \langle \mathcal{P}(U), \mathbf{R}, \varphi \rangle$, $u \in U$.

Proposition 1.1. *For any modifiers $\mathcal{F}_1$ and $\mathcal{F}_2 \in \mathbf{O}$ (when $\mathcal{F}_1^*$ and $\mathcal{F}_2^* \in \mathbf{O}^*$) and for any $\alpha \in \mathbf{W}$ it holds*

$$\mathcal{F}_1 \precsim \mathcal{F}_2 \Rightarrow \models \mathcal{F}_2(\alpha) \rightarrow \mathcal{F}_1(\alpha)$$

and

$$\mathcal{F}_1 \precsim \mathcal{F}_2 \Leftrightarrow \models \mathcal{F}_1^*(\alpha) \rightarrow \mathcal{F}_2^*(\alpha).$$ □

Proof. Let A_i be the universe of $\mathcal{F}_i, i = 1, 2$, corresponding to a wff α in an arbitrary *Ch*-sturcutre $\mathcal{U}$ and $\mathcal{F}_1, \mathcal{F}_2 \in \mathbf{O}$ such that $\Im \precsim \mathcal{F}_1 \precsim \mathcal{F}_2$.

(1°) From the assumption it follows $A_1 \subseteq A_2$ by Definition 1.4. Thus especially $\Im \precsim^{\mathcal{U}} \mathcal{F}_1 \precsim^{\mathcal{U}} \mathcal{F}_2$. If $\models^{\mathcal{U}} \mathcal{F}_2(\alpha)$ then $\models_u^{\mathcal{U}} \mathcal{F}_2(\alpha)$ for any $u \in A_2$. This is equivalent to the fact $\models_t^{\mathcal{U}} \alpha$ for all $t \in A_2$ such that $u\mathbf{R}t$ by Definition 1.5.(v). Because of $A_1 \subseteq A_2$, this holds also for all $t \in A_1$ such that $u\mathbf{R}t$, i.e. $\models_u^{\mathcal{U}} \mathcal{F}_1(\alpha)$. Thus $\models_u^{\mathcal{U}} \mathcal{F}_2(\alpha) \Rightarrow \models_u^{\mathcal{U}} \mathcal{F}_1(\alpha)$ from which it follows $\models_u^{\mathcal{U}} \mathcal{F}_2(\alpha) \rightarrow \mathcal{F}_1(\alpha)$ by Definition 1.5.(iii). The case not $\models_u^{\mathcal{U}} \mathcal{F}_2(\alpha)$ for some $u \in A_2$ is clear by *PC*. Because $\mathcal{U}$ and its world u were arbitrary, the result $\models \mathcal{F}_2(\alpha) \rightarrow \mathcal{F}_1(\alpha)$ is correct.

(2°) Suppose $\models \mathcal{F}_2(\beta) \rightarrow \mathcal{F}_1(\beta)$, $\beta \in \mathbf{W}$. From this it follows $\models \neg\mathcal{F}_1(\beta) \rightarrow \neg\mathcal{F}_2(\beta)$ by *PC* (and especially by the properties of '$\neg$' and '$\rightarrow$'). Substituting β by $\neg\alpha$ we get $\models \neg\mathcal{F}_1(\neg\alpha) \rightarrow \neg\mathcal{F}_2(\neg\alpha)$ which means that $\models \mathcal{F}_1^*(\alpha) \rightarrow \mathcal{F}_2^*(\alpha)$ holds by the definition of $\mathcal{F}^*$. ■

Corollary . *If $\mathcal{F} \in \mathbf{O}$ then for any $\alpha \in \mathbf{W}$ it holds*

$$\Im \precsim \Leftrightarrow \models \mathcal{F}(\alpha) \rightarrow \Im(\alpha)$$

and

$$\Im \precsim \mathcal{F} \Leftrightarrow \models \Im(\alpha) \rightarrow \mathcal{F}^*(\alpha)$$

and thus

$$\Im \precsim \mathcal{F} \Leftrightarrow \mathcal{F}(\alpha) \rightarrow \mathcal{F}^*(\alpha).$$ □

Proof. This follows directly from Proposition 1.1 by *PC*. ■

Proposition 1.2. *If $\mathcal{F} \in \mathbf{O}$ and $\mathcal{H} \in \mathbf{O}^*$, then for any wff $\alpha \in \mathbf{W}$ it holds*

$$\models \mathcal{F}(\alpha) \rightarrow \mathcal{H}(\alpha).$$ □

Proof. Let $\mathcal{F} \in \mathbf{O}, \mathcal{H} \in \mathbf{O}^*$ and $\alpha \in \mathbf{W}$. The case $\mathcal{H} = \mathcal{F}^*$ is clear by means of the corollary of Proposition 1.1. Let then $\mathcal{H} \precsim \mathcal{F}^*$. From this it follows $\models \mathcal{F}^*(\alpha) \rightarrow \mathcal{H}(\alpha)$

by Proposition 1.1, and from this and from the fact $\models \mathcal{F}(\alpha) \rightarrow \mathcal{F}^*(\alpha)$ it follows $\models \mathcal{F}(\alpha) \rightarrow \mathcal{H}(\alpha)$ by *PC*. After this case there is left only one situation, namely $\mathcal{F}^* \precsim \mathcal{H}$. Thus the both modifiers $\mathcal{F}^*$ and $\mathcal{H}$ belong to $\mathcal{O}^*$. Then for any wff $\beta \in \mathcal{W}$ it holds $\models \mathcal{H}(\beta) \rightarrow \mathcal{F}^*(\beta)$ by Proposition 1.1. From this it follows $\models \neg\mathcal{F}^*(\beta) \rightarrow \neg\mathcal{H}(\beta)$ by *PC*. This is the same as $\models \mathcal{F}(\neg\beta) \rightarrow \mathcal{H}^*(\neg\beta)$ by the definition of duals. Because $\mathcal{H}^* \in \mathcal{O}$ and $\mathcal{H} \in \mathcal{O}^*$, we have $\models \mathcal{H}^*(\neg\beta) \rightarrow \mathcal{H}(\neg\beta)$ by Proposition 1.1, and finally we get $\models \mathcal{F}(\alpha) \rightarrow \mathcal{H}(\alpha)$ when $\beta \equiv \neg\alpha$. This completes the proof. ■

Let U be an arbitrary *Ch*-structure. If $\models \alpha$ then $\models \mathcal{U}^u$ in any world u of $\mathcal{U}$ by the validity of α. Thus from the fact $\eta(\alpha, \mathcal{F}_j) = A \cup \{u \mid \text{ for all } t \in \varphi_\alpha(i_1, \ldots, i_k),\ t\mathbf{R}_j u\}$ it follows the condition

$$\models \alpha \Leftrightarrow \eta(\alpha, \mathcal{F}_j) = U \text{ for all } \mathcal{F}_j \in \mathcal{O}.$$

Proposition 1.3. *For any wffs $\alpha \in \mathcal{W}$ and modifiers $\mathcal{F} \in \mathcal{O}$*

$$\models \alpha \Rightarrow \models \mathcal{F}(\alpha). \qquad \square$$

Proof. Suppose $\Im \precsim \mathcal{F}$ be any modifier, $\mathcal{U}$ be any *Ch*-structure, the universe of $\mathcal{F}$ be $A \in \mathcal{P}(U)$ corresponding to α, and $\models \alpha$. Then for any *Ch*-structure $\mathcal{U}$ such that $A \in \mathcal{P}(U)$ and $\models^{\mathcal{U}}_u \alpha$ for all $u \in A$, which means that $A = \eta(\alpha, \mathcal{F}_j) = U$ for all $\mathcal{F}_j \in \mathcal{O}$. Thus $\models^{\mathcal{U}}_u \mathcal{F}(\alpha)$ holds for any *Ch*-structure $\mathcal{U}$ i.e. $\models \mathcal{F}_j(\alpha)$. ■

Proposition 1.4. *For any wffs $\alpha \in \mathcal{W}$ and the identity operator $\Im$ it holds*

$$\models \Im(\alpha) \leftrightarrow \alpha. \qquad \square$$

Proof. The result follows directly from Definition 1.5.(iv) by *PC*. ■

2. *Ch*-SYSTEMS

For the *proof-theory* we give a sound axiomatization for our system *LPC+ Ch*. In addition to the axiomatization of *LPC* we need in our proof-theoretical system a *characteristic axiom schemata* governing the logical properties of the modifier operators. Our axiomatisation for our system *LPC+ Ch* is as follows:

Axiom schemata of *Ch*. (i) All the *valid wffs of LPC* are axioms.
(ii) If $\mathcal{H}$, $\mathcal{F} \in \mathcal{O}$, and $\mathcal{H} \precsim \mathcal{F}$ then all $\alpha \in \mathcal{W}$

$$\mathcal{F}(\alpha) \rightarrow \mathcal{H}(\alpha) \qquad \textbf{(AxCh)}$$

in an axiom.
(iii) For all wffs $\alpha \in \mathcal{W}$ and for the identity operator $\Im \in \mathcal{O}$ it holds

$$\Im(\alpha) \leftrightarrow \alpha \qquad \textbf{(AxId)}$$

is an axiom.

We also adopt the following **inference rules**:

Modus ponens:

$$\alpha \to \beta,\ \alpha \vdash \beta \tag{MP}$$

Modified modus ponens:

$$\alpha \to \beta,\ \mathcal{F}(\alpha) \vdash \mathcal{F}(\beta) \tag{MMP}$$

where $\mathcal{F} \in \mathcal{O}$ is an arbitrary operator.

Rule of Substantiation: For wffs $\alpha \in \mathbf{W}$ and all substantiating operators $\mathcal{F} \in \mathcal{O}$,

$$\vdash \alpha \Rightarrow \vdash \mathcal{F}(\alpha) \tag{RS}$$

The rule (**RS**) can be illustrated intuitively by saying that a true fact remains true even if we try to substantiate it.

So, a *Ch-system* is any non-empty set X such that the valid wffs of *LPC*, (**AxCh**), and (**AxId**) are included in X, and X is closed under (**MP**),(**MMP**) and (**RS**).

We have to prove that all the theorems (i.e. wffs $\alpha \in \mathbf{W}$ such that $\vdash \alpha$) of our *Ch*-system are valid. this property is called *soundness*. The proving method is such that first we prove that the axiom schemata of the *Ch*-system are valid, and secondly, the inference rules preserve validity.

Proposition 2.1. (soundness). *For any $\alpha \in \mathbf{W}$ it holds*

$$\vdash \alpha \Rightarrow \models \alpha. \qquad \square$$

Proof. (1°) The validity of the valid wffs of W is clear by *LPC* and Definition 3.1.
(2°) The validity of (**AxCh**) follows directly from Propositions 1.1 and 1.2.
(3°) The validity of (**AxId**) follows directly from Proposition 1.4.
(4°) The rule (**MP**) is clear by *LPC*.
(5°) Consider the rule (**MMP**). Let $\mathcal{U}$ be an arbitrary *Ch*-structure, and let $\models_u^{\mathcal{U}} \alpha \to \beta$ and $\models_u^{\mathcal{U}} \mathcal{F}(\alpha)$. Suppose first that $\Im \precsim \mathcal{F}$, and let the universe of $\mathcal{F}$ be $A \in \mathcal{P}(U)$ for $\alpha \in \mathbf{W}$, and $u \in A$. We have

$$\models_u^{\mathcal{U}} \mathcal{F}(\alpha) \Rightarrow \text{ for all } t \in A \text{ and } u\mathbf{R}t \text{ it follows } \models_t^{\mathcal{U}} \alpha.$$

Thus $\models_u^{\mathcal{U}} \alpha$ for all $u \in A$ by Definition 1.3.(v). By Definition 1.3.(iii) it follows from this that $\models_u^{\mathcal{U}} \beta$ for all $u \in A$, i.e. for all $t \in A$ and $u\mathbf{R}t$ it follows $\models_t^{\mathcal{U}} \beta$, which means that $\models_u^{\mathcal{U}} \mathcal{F}(\beta)$. Because $\mathcal{U}$ was chosen arbitrarily, we have $\models \mathcal{F}(\beta)$. Suppose then that $\mathcal{F} \precsim \Im$, and let the universe of $\mathcal{F}$ be $A \in \mathcal{F}(U)$, and $u \in A$. We have

$$\models_u^{\mathcal{U}} \mathcal{F}(\alpha) \Rightarrow \text{ there exists } t \in A \text{ such that } u\mathbf{R}t \text{ and } \models_t^{\mathcal{U}} \alpha.$$

Thus $\models_u^{\mathcal{U}} \alpha$ for some $u \in A$ by Definition 1.3.(vi). By Definition 1.3.(iii) it follows from this that $\models_u^{\mathcal{U}} \beta$ for those $u \in A$, as $\models_u^{\mathcal{U}} \alpha$, i.e. there exists $t \in A$ such that $u\mathbf{R}t$ and $\models_u^{\mathcal{U}} \beta$, which means that $\models_u^{\mathcal{U}} \mathcal{F}(\beta)$. Because $\mathcal{U}$ was chosen arbitrarily, we have $\models^{\mathcal{F}} (\beta)$.
(6°) The rule (**RS**) is clear by Proposition 1.3. ■

3. SOME COMMENTS AND IDEAS

In the above described Kripke-semantics of $LPC+Ch$ there are some interesting questions. For example there are no special descriptions about the accessibility relation **R**. It can be mainly supposed that **R** is a binary relation on U. It also very natural that we can get different *Ch*-systems, if we give different properties to **R** like in modal logic. I think **R** should be at least reflexive in U for the semantico-syntactical completeness of the system, i.e. the set of all valid formulae of the *Ch*-language is exactly the set of the theorems of the *Ch*-system. However, this question is left open in this paper.

There are also interesting possibilities for fussy extensions of the *Ch*-language. These come into the question, if we extend the power set $\mathcal{F}(U)$ of the universe of U such that we include fussy subsets of U in $\mathcal{P}(U)$. Also these considerations are not made in this paper.

One possibility to apply the system $LPC+Ch$ is the *logic of hedges* (see e.g. [3] and [7], which papers give some ideas for that). In that case a modifier is an operator $\mathcal{F} : [0,1] \rightarrow \{0,1\}$. The idea is as follows: If we interpret the function $\mu : [0,1]^n \rightarrow [0,1] (n = 1,2,\ldots)$ as truth-function, and for some expression α it holds $\mu(\alpha) = r$ $(0 < r < 1)$, we can operate with a weakening operator $\mathcal{H}$ on it such that we get $\mu(\mathcal{H}(\alpha)) = 1$. After manipulating all the expressions like α above we can apply the system $LPC+Ch$. After getting the conclusion we use the inverse operators $\mathcal{F}^{-1}$ for evaluating the value $\mu(\beta)$ for the conclusion β.

Example. Consider an inference of the form

0.7	α
0.8	$\alpha \rightarrow \beta$
?	β

In the first premise we apply modifiers as hedges so that we operate the wff α by such a weakening modifier $\mathcal{H}$ that the truth value of $\mathcal{H}(\alpha)$ becomes 1, i.e. the modifier $\mathcal{H}$ is a mapping $\mathcal{H} : [0,1] \rightarrow \{0,1\}$ which maps 0.7 into 1. In the second premise the truth value of β is less than that of α, because the truth value of the implication is less than 1. We now operate the wff β by such a weakening modifier $\mathcal{V}$ that the truth value of $\mathcal{V}(\beta)$ becomes greater than that of α. Then by means of the Calois connection,the implication $\alpha \rightarrow \mathcal{V}(\beta)$ has the truth value 1. Now we get the inference

1	$\mathcal{H}(\alpha)$
1	$\alpha \rightarrow \mathcal{V}(\beta)$
1	$\mathcal{H}\mathcal{V}(\beta)$

by **MMP**. Because in the conclusion β is operated by a chain of two weakening modifiers, we would need the operation $\mathcal{H}\mathcal{V}^{-1}$ for getting the truth value of β which in this case must be less that that of any premise in the original inference, i.e. less than 0.7, say 0.5 or 0.6. The definition of the inverse modifiers $\mathcal{F}^{-1}$ are not clear yet, and there will appear some difficulties with them.

This thing is still just an idea about how to use linguistic approximations in *LPC+Ch* to create fuzzy truth values for conclusions when we know those values for the premises. One of the main questions is, how well does the system *LPC+Ch* fit together with hedges. There are some interpretational problems which need additional investigations. These problems are not surprising because the relationship between formal languages and the natural language is always problematic.

REFERENCES

[1] G.E. Hunghes, M.J. Cresswell, *An Introduction to Modal Logic* (Methuen and Co., reprinted 1985)

[2] S.A. Kripke, Semantical considerations on modal logic, *Acta Philosophica Fennica*, Fasc, XVI. 1963.

[3] G. Lakoff, Hedges: A study in meaning criteria and the logic of fuzzy concepts, *The Journal of Philosophical Logic*, **2**, 1973.

[4] E.J. Lemmon, An Introduction to Modal Logic *American Philosophical Quarterly*, Monograph No. 11 (ed. by Segerberg), Oxford, 1977.

[5] A. Margaris, *First Order Mathematical Logic* (Blaisdell, 1967)

[6] J.K. Mattila, Proof-theoretical aspects of fuzzy logic, In:Bandler and Kandel (eds), *Recent Developments in the Theory and Applications of Fuzzy Sets* (Proceedings of North American Fuzzy Information Processing Siciety NAFIPS'86 Conference, New Orleans, Louisiana, June 2-4, 1986., 386-398.

[7] J.K. Mattila, *Calculus of modifier operators in fuzzy logic*, Publications of the Institute for Applied Mathematics No. 14, University of Turku, Turku, Finland, 1989.

[8] J.K. Mattila, *The construction of a propositional modifier logic,* Research Report No 21, Lappeenranta University of Technology, Department of Information Technology, Lappeenranta, 1990.