# FACTS IN PLACE/TRANSITION-NETS WITH UNRESTRICTED CAPACITIES

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## 1. Introduction

By a fact, as introduced by Petri [Pe], an additional system specification or assertion on a system behaviour is formulated. Descriptions by facts are attractive, since they are formulated within the language of net theory [GLT]. A fact in a place/transition net (PT-net) has the syntactic form of a transitional element, with the semantical property that only such markings are considered to be correct states of the system, where this transition has *not* concession. The fact of Fig. 1.1, for instance, specifies the condition, that in no reachable marking both $p_1$ and $p_2$ contain a token. This fact might belong to a system of reader and writer processes, where the number of tokens in $p_1$ and $p_2$ indicates how many readers and writers, respectively, are in the critical section. Together with the capacities 5 and 1 this fact represents the specification: "Writers are in mutual exclusion with all readers and all other writers. Furthermore at most 5 readers have access."

In [GeLa] a systematic way for implementing fact specifications by PT-nets is proposed under the restriction, that all places have finite capacities. We here will show a natural way to extend this to the general case. Then it will be also possible to realize a fact like Fig. 1.2, where $p_1$ has not finite capacity. Consequently in this example the simultanous access of readers is not bounded.

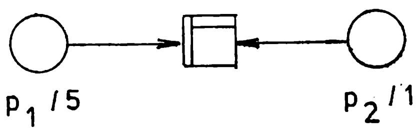



*Fig. 1.1*

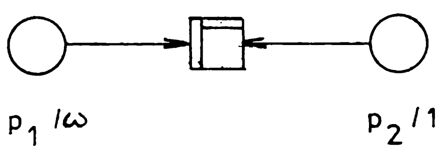



*Fig. 1.2*

## 2. Nets and facts

We first give a formal definition of PT-nets [JaVa] and facts in PT-nets [GeLa].

*Definition 2.1.* A *place/transition net (PT-net)* is given by $N=(S, T, F, K, W, M_0)$ where:

*a)* $S$ and $T$ are finite disjoint sets of places and transitions, respectively,
*b)* $F \subseteq (S \times T) \cup (T \times S)$ is the flow relation,
*c)* $K: S \to \mathbf{N} \cup \{\omega\}$ assigns to each place a finite $(K(s) \in \mathbf{N})$ or infinite $(K(s)=\omega)$ capacity,
*d)* $W: F \to \mathbf{N}$ assigns to each arc its multiplicity,
*e)* $M_0: S \to \mathbf{N}$ is the initial marking.

To simplify the definition of the firing rule we extend $W$ to $\hat{W}: (S \times T) \cup \cup (T \times S) \to \mathbf{N}$ by $\hat{W}(x, y) := \text{if } (x, y) \in F \text{ then } W(x, y) \text{ else } 0$.

*Definition 2.2.* Let be $t \in T$ a transition and $M: S \to \mathbf{N}$ a marking. Define a map $M': S \to \mathbf{Z}$ by

$$M'(s) := M(s) - \hat{W}(s, t) + \hat{W}(t, s).$$

Then $t$ *has concession* or *can fire* if $M(s) \geq \hat{W}(s, t)$ and $M'(s) \leq K(s)$ for all $s \in S$. *$t$ fires $M$ to $M_1$* : $M[t > M_1$ iff $t$ has concession in $M$ and $M_1 = M'$. $[M> := \{M_1 | M[> M_1\}$ is the set of markings *reachable* from $M$, where $[>$ is the transitive and reflexive closure of $\{(M, M_1) | \exists t: M[t > M_1\}$.

By this firing rule we have the following property.

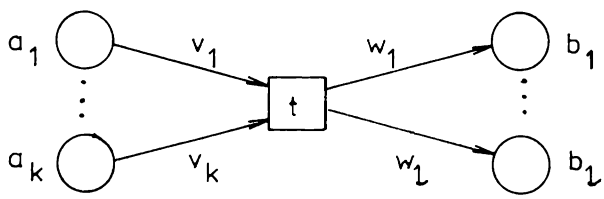



*Fig. 2.1*

**Proposition 2.3.** [GeLa] *Let be $t \in T$ the transition of Fig. 2.1* and $^\bullet t := := \{a_1, \ldots, a_k\}$, $t^\bullet = \{b_1, \ldots, b_l\}$ *and*

$$\alpha_i := (M(a_i) < v_i) \qquad \textit{for} \quad 1 \leq i \leq k,$$
$$\beta_j := (M(b_j) + w_j > K(b_j)) \quad \textit{for} \quad 1 \leq j \leq l.$$

*Then $t$ is dead, i.e. can fire in no $M \in [M_0>$ iff for all $M \in [M_0>$ :*

(I) $$\alpha_1 \vee \ldots \vee \alpha_k \vee \beta_1 \vee \ldots \vee \beta_l.$$

*Definition 2.4.* A transition $t \in T$ is a *fact* for a marking $M$, if (I) of 2.3 holds. It is a fact of the net $N$ if it is a fact for all $M \in [M_0>$.

By the assumption $\omega \geq n$ for all $n \in \mathbf{N}$, $\beta_j$ becomes identically false if $K(b_j) = \omega$ and can then be omitted in (I).

## 3. The realization of facts

By a realization (or implementation) of a fact $t$ we understand (as in [GeLa]) a net, for which $t$ is a fact and which has a maximal set of reachable markings with this property. The last condition assures that the realization contains no undesired side-effects.

*Definition 3.1.* Let $t$ be a transition as in 2.3. A *realization* of $t$ is a net $N=(S, T, F, K, W, M_0)$ such that $^\bullet t \cup t^\bullet \subseteq S$ and ($M \in [M_0>$ iff $t$ is a fact for $M$).

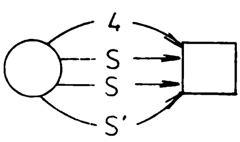
If for all $s \in S: K(s) < \omega$ the construction of a realization for $t$ of Fig. 2.1 in [GeLa] proceeds as follows. First for each arc $A_i=(a_i, t)$ $(1 \le i \le k)$ and $B_j=(t, b_j)$ $(1 \le j \le l)$ PT-nets $NA_i$ and $NB_j$, respectively, are built. These nets have special places $r_i$ and $s_j$, respectively. The emptiness of this place indicates that condition $\alpha_i$ and $\beta_j$, respectively, is true. For an instance of $NA_i$ consider the net in Fig. 3.3 and omit transitions "in" and "out". Realize that $r_i := r$ has the required properties. Hence, to obtain a realization of $t$, these nets are connected by a new place $d$ in such a way, that the invariant

$$(\mathrm{i}_0) \qquad M(r_1)+\ldots+M(r_k)+M(s_1)+\ldots+M(s_l)+M(d)=k+l-1$$

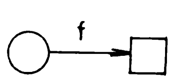
is satisfied for all $M \in [M_0>$. By this at least one of the conditions $\alpha_i$ or $\beta_j$ is true at any time.

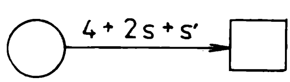
We now extend this construction for $K(s)=\omega$. As proved by many authors and as discussed in [Va81] there cannot exist a realization by a PT-net for the fact in Fig. 1.2. Therefore we have to leave the framework of PT-nets. But we will do it in such a way, that the solution is as close as possible to the one described above, i.e. it will have the same graphical representation and the same invariants. To this end we will formulate the solution in the framework of SM-nets, which are a natural extension of PT-nets [Va78].

In a SM-net the multiplicity of an arc can also be the name of an arbitrary place of the net. In an actual marking $M$ this name $s$ is replaced (evaluated) by its token number $M(s)$ and then the net fires as a PT-net. Since the inscriptions of the net are changed by the dynamical behaviour of the net, SM-nets are called selfmodifying nets. To allow multiple arcs like:




we use the notation of formal sums:

or a map $f$: 

where 


$f: \{e, s, s', s''\} \rightarrow \mathbf{N}$ with

$f(e)=4$, $f(s)=2$, $f(s')=1$, $f(s'')=0$. In the formal definition we will use the last representation. An example of the firing rule is given in Fig. 3.1 (where all capacities are $\omega$).

*Definition 3.2.* A *selfmodifying net (SM-net)* $N=(S, T, F, K, W, M_0)$ is defined like a PT-net in 2.1, with the difference of:

d) $W: F \to Map\ (S \cup \{e\}, \mathbf{N})$ assigns to each arc its multiplicity. Map $(X, Y)$ is the set of all maps from $X$ to $Y$.

Every PT-net $N=(S, T, F, K, W, M_0)$ is isomorphic to the SM-net $N_1=(S, T, F, K, W_1, M_0)$, where for all $s \in S$: $W_1(x, y)(e)=W(x, y)$ and $W_1(x, y)(s)=0$.

Let be $0 \in Map\ (S \cup \{e\}, \mathbf{N})$ the zero-map $x \mapsto 0$. Then $\hat{W}$ is defined as for PT-nets.

*Definition 3.3.* For a marking $M$ and a SM-net multiplicity-map $W$ we define the *evaluation of W at M* $W(M): F \to \mathbf{N}$ by

$$W(M)(x, y) := W(x, y)(e) + \sum_{s \in S} W(x, y)(s)\,(M(s)).$$

Then for a SM-net $N=(S, T, F, K, W, M_0)$ and a marking $M$ a PT-net $N(M)=(S, T, F, K, W(M), M_0)$ is defined. We define the firing rule for the SM-net by: $M[t> M'$ in $N$ iff $M[t> M'$ in $N(M)$.

Concurrent firing of two transitions is possible if they have disjoint input and output places (as in the case of PT-nets) and disjoint sets of places as multiplicities on incidenting arcs.

We now proceed in the construction of a realization for the fact in Fig. 2.1. If $K(a_i)$ or $K(b_j)$ belongs to $\mathbf{N}$, we construct the net $NA_i$ or $NB_j$ as in [GeLa]. If $K(b_j)=\omega$ the proposition $\beta_j$ is identically false. Hence, we can equivalently omit the arrow $(t, b_j)$ in this case. What is left, is to construct the net $NA_i$ for the case $K(a_i)=\omega$ in an appropriate way. To this end we consider an arbitrary such arc as in Fig. 3.2 with $K(p)=\omega$.

We start with the net from Fig. 3.2 of [GeLa] with $K(p)=k$. (This is the net in our Fig. 3.3, when transitions "in" and "out" are omitted.) To extend this net from the fixed capacity $k \in \mathbf{N}$ to unbounded capacity, we introduce a new place $k$ with $M_0(k)=k$ tokens. Then interpreting the letter $k$ on arcs not as a constant $k \in \mathbf{N}$ but as the name $k \in S$ of a place, we obtain a SM-net with the same behaviour as before (the arc labelled by $k-m+1$ stands for one arc labelled $k+1$ and a second with *inverted* direction, which is labelled $m$.) Now we want to change the net in such a way that the value of $k$ can be changed dynamically.

This can be done by adding new transitions "in" and "out" and arrows $(in, k)$ and $(k, out)$ (see Fig. 3.3). The changing of the 'actual' capacity $M(k)$ of $p$ should be required to be independent of the behaviour of the rest of the net. By this we mean, that for arbitrary $t \in T-\{in, out\}$ a firing of the two transitions "*t, in*" ("*out, t*") in this order should have the same effect as "*in, t*" ("*t, out*"). This is no problem with the exeption of $t=C$ and $t=D$, where the multiplicities contain $k$. For instance in the case of $t=C$ a firings of "*t, in*" brings $k-m+1$ tokens to the place 3, whereas "*in, t*" brings $(k+1)-m+1$. This must be compensated by additional selfmodifying arcs $(in, 3)$ and $(3, out)$ with multiplicity $r$.

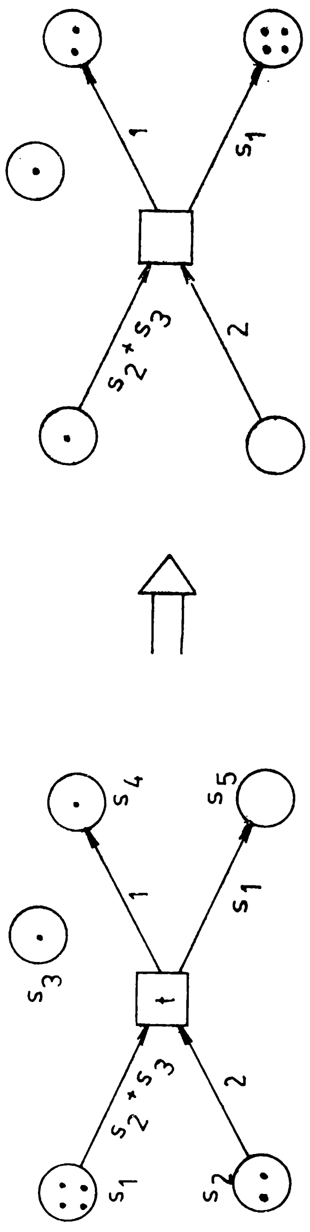



*Fig. 3.1*


*Fig. 3.2*

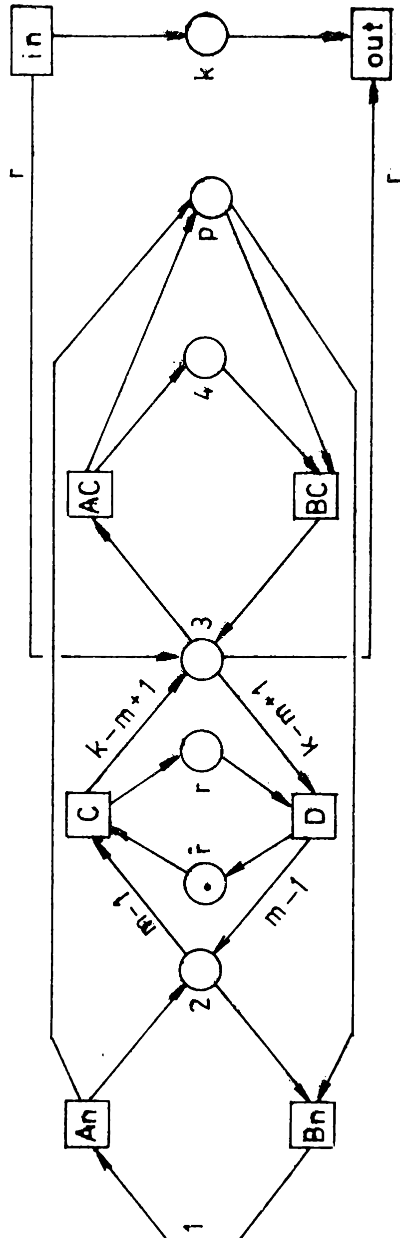



*Fig. 3.3*

Now the net NA works correctly, i.e. the markings on $p$ and $r$ have the possible values (0,0), (1,0), (2,0), ..., $(m-1,0)$, $(m-1,1)$, $(m, 1)$, $(m+1,1)$, ... i.e. $M(r)=0$ implies that $t$ is a fact. The construction of the realization can be completed as described above.

## 4. Bilinear invariants

It becomes now necessary to sharpen our analysis of the SM-net is Fig. 3.3 by proofs using invariants. These should be similar as those given in [GeLa]. Replacing $k$ by $M(k)$ we obtain for all $M\in[M_0>$ the invariant equations:

$$(\mathrm{i}_1) \qquad M(1)+M(2)+(m-1)M(r)=m-1,$$

$$(\mathrm{i}_2) \qquad M(p)+M(1)-M(4)=m-1,$$

$$(\mathrm{i}_3) \qquad M(k)M(1)+M(k)M(2)+(m-1)\big(M(3)+ +M(4)-M(1)-M(2)-M(k)\big)=-(m-1)^2,$$

$$(\mathrm{i}_4) \qquad M(3)+M(4)-M(k)M(r)+(m-1)M(r)=0,$$

$$(\mathrm{i}_5) \qquad M(r)+M(\hat{r})=1.$$

Since these equations are bilinear forms, we call them *bilinear invariants*. They can be used to prove as in [GeLa]:

**Proposition 4.1.** *For all* $M\in[M_0>$:

*a)* $M(r)=0\Rightarrow M(p)<m$ (*i.e.* $\alpha=true$),

*b)* $M(r)=0\Leftarrow M(p)<m-1$ (*i.e.* $\alpha=true\wedge M(p)\neq m-1$).

This gives the desired property of the indicator place $r$. As mentionned in [Va81] bilinear invariants have a representation by matrices (like bilinear forms), which can be computed from the SM-net. By solving a linear system of integer valued equations, the method applied here, namely deriving bilinear invariants from linear invariants, is more efficient if possible.

## 5. Special solutions and examples

For the special case of the fact in Fig. 5.1 describing mutual exclusion with unbounded capacities we obtain the realization of Fig. 5.2.

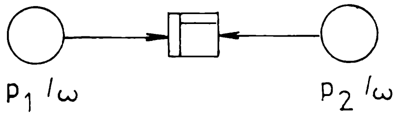



*Fig. 5.1*

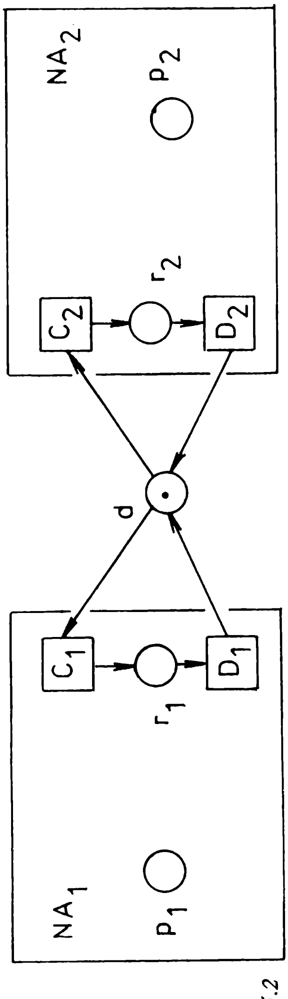



*Fig. 5.2*

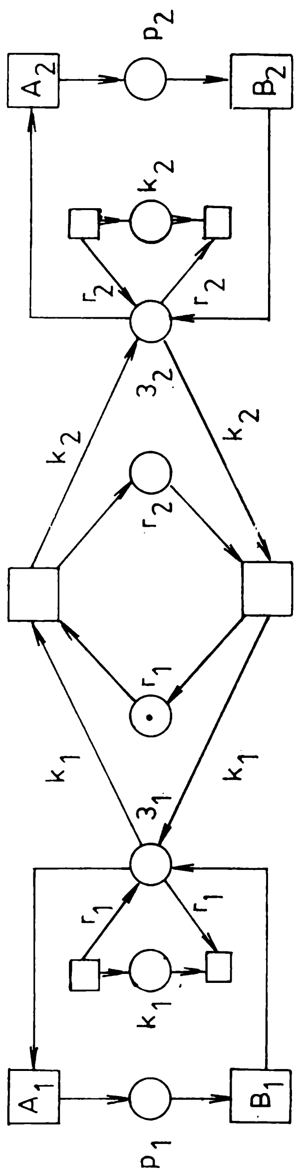



*Fig. 5.3*

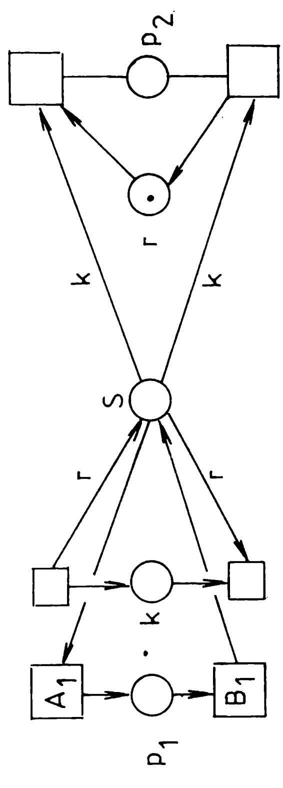



*Fig. 5.4*

This net can be contracted to the net of Fig. 5.3.

From the invariants $(i_0)-(i_5)$ we obtain:

$$(i_6) \qquad M(p_1)+M(3_1) = M(k_1)M(r_1),$$

$$(i_7) \qquad M(p_2)+M(3_2) = M(k_2)M(r_2),$$

$$(i_8) \qquad M(r_1)+M(r_2) = 1.$$

By these equations the following bilinear invariant can be proved, which expresses the mutual exclusion of $p_1$ and $p_2$.

$$(i_9) \qquad M(p_1)M(p_2) = 0.$$

To obtain a realization of the fact in Fig. 1.2 describing the readers/writers problem, we have to restrict $k_2$ in Fig. 5.3 by the capacity $K(k_2) = 1$. By further contraction we then obtain the realization of Fig. 5.4. For this SM-net the following bilinear invariant holds:

$$(i_{10}) \qquad M(p_1)+M(s)+M(k)M(p_2) = M(k).$$

## 6. Stepwise realizations

In a practical implementation a fact can be given not in isolation but within the framework of a "partial realization". For instance in the case of the unbounded readers/writers problem we could start with the partial realization of Fig. 6.1. "in" and "out" perform the arrival and exit, respectively, of processes. On $l_1$ or $l_2$ they do some work on local data. With the firing of $A_1$ and $A_2$ they give the information whether they act as readers or writers. $k$ counts the actual number of processes in the system. The net has the following (linear) invariant:

$$(i_{11}) \qquad M(p_1)+M(p_2)+M(l_1)+M(l_2) = M(k).$$

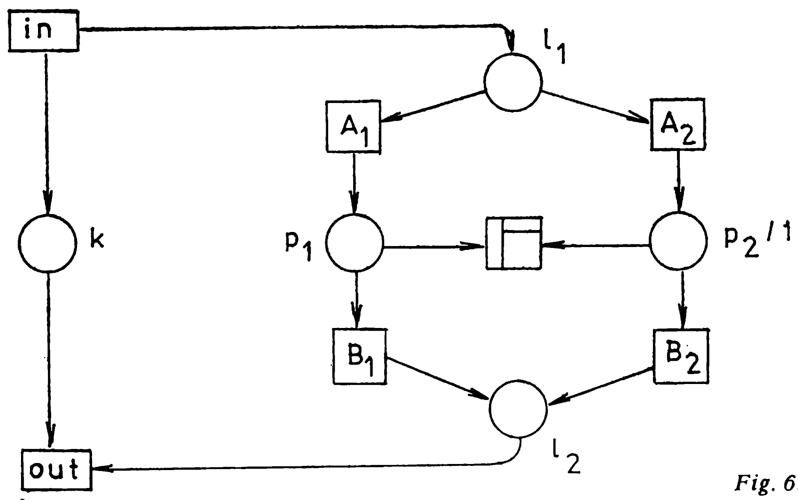



*Fig. 6.1*

By realizing the fact as in Fig. 5.4 we obtain the "complete" realization in Fig. 6.2. It satisfies the invariants $(i_{10})$ and $(i_{11})$.

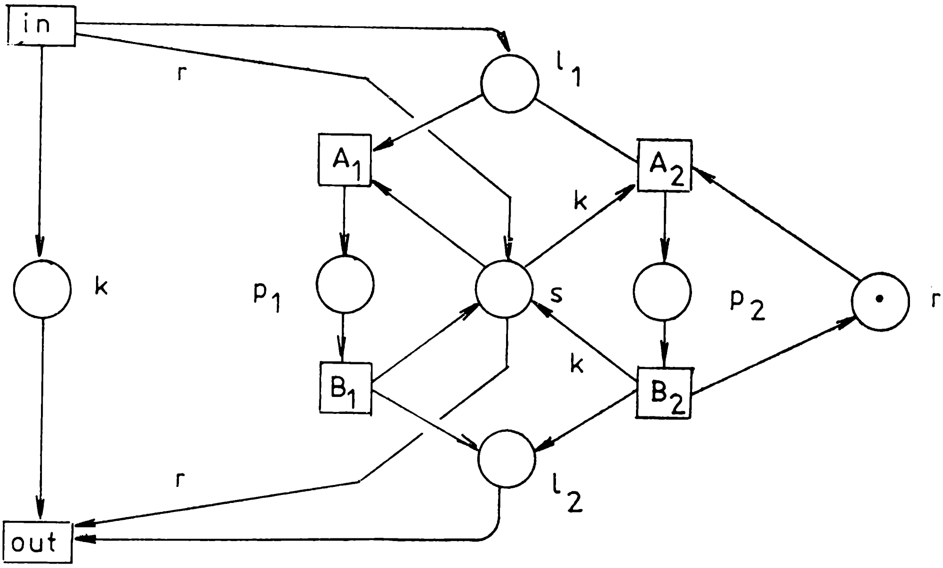



*Fig. 6.2*

This solution has been described also in [Va81]. By invariants $(i_{10})$ and $(i_{11})$ many interesting properties can be proved in the same way as for the bounded version (see [Va81]).

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