REDUCTION OF THE ON HAND INVENTORY BY USING TRANSACTION REPORTING SYSTEM

LÁSZLÓ GERENCSÉR

(Received July 27, 1973)

Introduction

The reduction of the on hand inventory is an important task, which is not sufficiently investigated in the usual models. The wellknown inventory models allow operations only which can reduce the on hand inventory at the price of increasing some other factor, say shortage. However if a change of models is realizable the mentioned difficulty may not arise. In this paper we show, that if we change an order-up-to S policy for an r, Q policy (which is a transaction reporting system) in a simple way we can reduce both the average on-hand inventory and the average shortage without changing the frequency of orders and the reduction is also estimated. Thus we avoid the using of costs in the operation, the improvement is reached for the physical processes themselves. Similar results are given in [1]. A thorough exposition of the models is given in [2].

1. Description of the models. We keep stock of a single item, which is demanded continuously. The amount demanded in the time interval $(0, t)$ is $\alpha(t)$, where $\alpha(t)$ is a stochastic process. We approximate $\alpha(t)$ by a Wiener process, whose parameters are m, σ. Thus the distribution of $\alpha(t)$ is $\Phi\left(\frac{x-mt}{\sigma\sqrt{t}}\right)$, where $\Phi(y)$ is the standardized normal distribution function. The replenishment of the stock is done by placing orders. The shipment of an order takes place time τ later, where τ is constant.

The S model is a periodic review model. The length of the period is T, orders are placed at times $O, T, 2T, \ldots$. The inventory position after ordering is S. A plausible way of saying it is the following: each time order the consumption of the previous period.

The (r, Q) model may be used when the changes of the inventory are reported continuously. Whenever the inventory level reaches a critical value r, immediately an order is placed for a lot Q. Thus the inventory position after each ordering is the same $r+Q$, but clearly the r, Q model is more sensitive to the fluctuations of the demand than the S-model.

Though the operations which will be given are independent of the prices, prices do exist, and now introduce three of them: A the price of placing and receiving an order IC unit time holding cost of unit quantity, $\hat{p}$ unit time shortage cost of unit quantity. The unit time average total costs are denoted by $K_1(S)$ and $K_2(r, Q)$. The unit time average on-hand inventory and shortage will be denoted by D and B, respectively, using correspondent subscripts.

To get the formula of $K_1(S)$ for the S-model we first recite, that

$$D_1 = B_1 + S - m\tau - \frac{mT}{2} \tag{1.1}$$

from which we easily get

$$K_1(S) = \frac{A}{T} + IC\left(S - m\tau - \frac{mT}{2}\right) + (IC + \hat{p})B_1. \tag{1.2}$$

For later purposes we give the formula of B_1 detailed. First we can write

$$B_1(t) = \int_{\tau}^{\tau+T} B_1(t)\,dt \tag{1.3}$$

where $B_1(t)$ is the expected shortage occuring at time t. Its formula is the following

$$B_1(t) = \int_{\sigma}^{\infty} (y - S)\frac{1}{\sigma\sqrt{t}}\ \varphi\left(\frac{y - mt}{\sigma\sqrt{t}}\right) dy. \tag{1.4}$$

In case of the (r, Q) model we have the similar identity

$$D_2 = B_2 + r + \frac{Q}{2} - m\tau. \tag{1.5}$$

From this we easily get

$$K_2(r, Q) = \frac{mA}{Q} + IC\left(r + \frac{Q}{2} - mt\right) + (IC + \hat{p})\,B_2 \tag{1.6}$$

The formula of B_2 is

$$B_2 = \frac{1}{Q}\int_{r}^{r+C} B_2(x)\,dx \tag{1.7}$$

where $B_2(x)$ is the expected shortage after time τ, when the initial inventory position is x. $B_2(x)$ is given by

$$B_2(x) = \int_{x}^{\infty} (y - x)\frac{1}{\sigma\sqrt{\tau}}\ \varphi\left(\frac{y - m\tau}{\sigma\sqrt{\tau}}\right) dy. \tag{1.8}$$

2. Correspondence between the two models. Suppose that we follow an order-up-to S policy. Suppose that the expected value of the inventory position just before placing an order is $S-mT$, and the amount that is ordered has the expectation mT. If we can change to a transaction reporting system, it is therefore natural to put

$$r = S-mT \text{ and } Q = mT \tag{2.1}$$

A look at the formulas of the previous point will show us, that this is in fact a fortunate choice. Substract equality (1.5) from equality (1.1) then we get

$$D_1-D_2 = B_1-B_2 \tag{2.2}$$

We see that D and B change simultaneously, hence it will suffice to prove B_2 is less than B_1. Note that the frequency of orders remained unchanged.

For the sake of completeness we write out the difference of the total costs, which we shall denote shortly by K_1, K_2. We have

$$K_1-K_2 = (IC+\hat{p})(B_1-B_2). \tag{2.3}$$

We are now confronted with the task of proving that

$$B_1 \geq B_2. \tag{2.4}$$

To prove this we give a further correspondence between the two models. We shall let time correspond to inventory position in the following way. Let s be a parameter which varies from O to T. The corresponding values are then

$$\tau+s \text{ and } S-ms \tag{2.5}$$

The value of $B_1(t)$ and $B_2(x)$ at these arguments will be denoted by $\overline{B}_1(s)$ and $\overline{B}_2(s)$. Our next step will be to prove that

$$\overline{B}_1(s) \geq \overline{B}_2(s) \tag{2.6}$$

3. Positive integral of a normal variable. Let us remind the meaning of $\overline{B}_1(s)$. Defining the variable

$$\xi_1(s) = \alpha(\tau+s)-S \tag{3.1}$$

$\overline{B}_1(s)$ is nothing but the expected value of the positive part of this variable, i.e.

$$\overline{B}_1(s) = E(\xi_1^+(s)). \tag{3.2}$$

For the second case introduce the variable

$$\xi_2(s) = \alpha(\tau)+ms-S \tag{3.3}$$

then clearly

$$\overline{B}_2(s) = E(\xi_2^+(s)). \tag{3.4}$$

Now notice, that

$$E(\xi_1) = E(\xi_2) \tag{3.5}$$

but

$$D^2(\xi_1) = D^2(\xi_2), \tag{3.6}$$

the former being equal to $\sigma^2(\tau+s)$ the latter to $\sigma^2\tau$.

Speaking more generally take a normally distributed random variable with parameters m, σ. For our purposes it is enough to whow that $E(\xi^+)$ is monoton increasing in σ when m is fixed. The formula of $E(\xi^+)$ is given by

$$R(m, \sigma) = \int_{\sigma}^{\infty} x \frac{1}{\sigma} \varphi\left(\frac{x-m}{\sigma}\right) dx. \tag{3.7}$$

This integral can be evaluated explicitly and we get

$$R(m, \sigma) = \sigma\varphi\left(\frac{m}{\sigma}\right) + m\Phi\left(\frac{m}{\sigma}\right). \tag{3.8}$$

The derivation with respect to σ yields

$$\frac{\partial R(m, \sigma)}{\partial \sigma} = \varphi\left(\frac{m}{\sigma}\right) \tag{3.9}$$

which proves our assertion.

Thus we completely proved that the transaction reporting system has the desired properties. We may give also estinations taking into account the convexity of $R(m, \sigma)$ in σ. Convexity is clear from (3.9) After easy calculations we may give the following rough estimation

$$B_1 \geq B_2 + \sigma \cdot T \cdot \gamma, \tag{3.10}$$

$$\gamma = \varphi\left(\frac{S - m\tau}{\sigma\sqrt{\tau}}\right) \frac{1}{4\sqrt{\tau + T}}. \tag{3.11}$$

This is valid under the assumption that in the current S model on-hand-inventory is greater than shortage. We present this estimation because of its simplicity, though other more refined estimations are available.

Bibliography

[1] *Gerencsér L.*, Reduction of the on hand inventory on the same level of reliability (to appear)

[2] *Hadley G* and *Whitin T.*, Analysis of inventory systems, Prentice Hall.